Phase Jitter

Phase Noise and Voltage Controlled Crystal Oscillators

David Chandler, VCXO / Hybrid Design Engineer

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Phase Jitter – Phase Noise and Voltage Controlled Crystal Oscillators.

Short-term frequency instabilities, seen in the time domain as jitter, can cause problems in both analog and digital signals. As system operating frequencies have increased, these instabilities have gained increasing importance, because their relative size to the total period length is larger. The instabilities can eventually cause slips or missed signals that result in loss of data. Figure 1 shows a square wave with jitter compared to an ideal signal at the same long-term frequency.

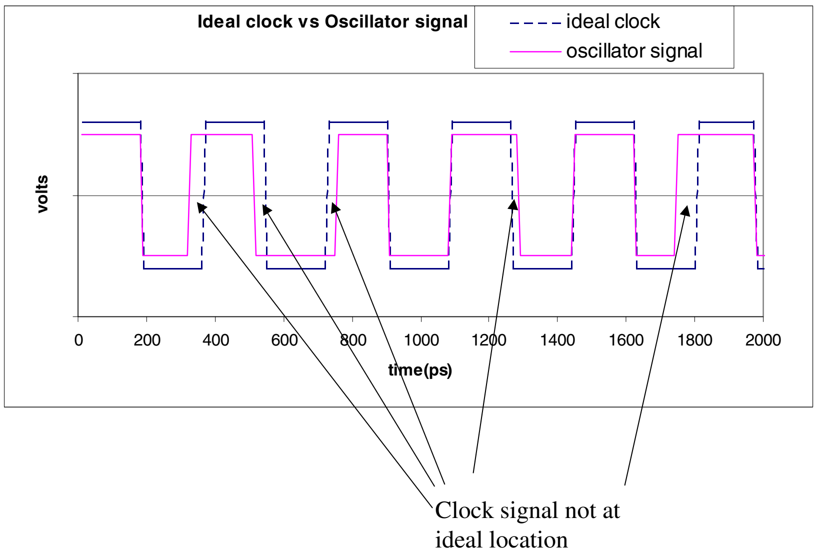


Figure 1: clock signal with short-term instabilities.

Since all frequency control devices will have some level of short-term instability, it is necessary to have quantifiable measures. Phase Noise, Allan Variance, Phase Jitter, Wander, Time Interval Error, Cycle-to-Cycle Jitter, Period Jitter, RMS vs. Peak-to-Peak vs. 1σ, and Bandwidth Limited Jitter are all terms used for the characterization of short term frequency instabilities. This application note discusses three time domain measurement techniques, and two frequency domain measurement techniques. Through these discussions, the difference between bandwidth-limited jitter and unspecified bandwidth measurements is illustrated.

Visual Representation of Short-Term Instability.

In order to understand the characterizations, it is easiest to discuss instabilities for a sine wave in the time domain, and then extend the discussion to square waves and the frequency domain.

An ideal sinusoidal voltage source can be characterized by the equation

$V(t)=Vp\sin(2\pi f_o t)$ equation 1

Vp = peak amplitude
f_o = *nominal frequency of the oscillator.*

The sinusoidal signal in figure 2 that varies from –2.5 to 2.5 Volts, with a nominal frequency of 1MHz would have

$V(t)=2.5\sin(2\pi 1x10^6 t)$ equation 2

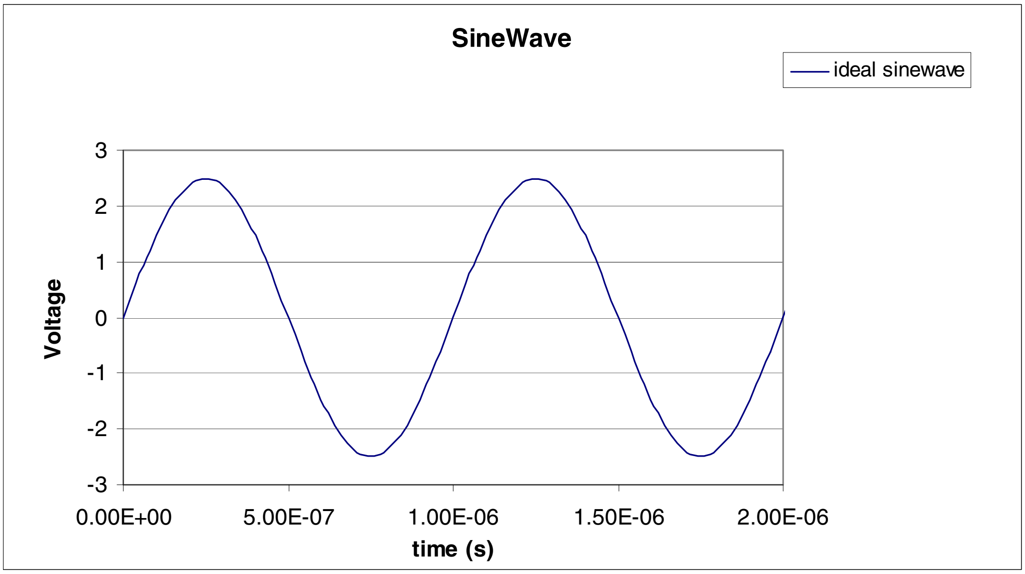


Figure 2: Ideal sinewave at 1 MHz with 2.5 Volt peak amplitude.

Noise in the signal can be broken down into two separate attributes:
1 – The peak values of the signal vary in time. This is amplitude modulation noise (AM).
2 – The point the signal crosses a reference voltage varies from its ideal location. This is phase modulation noise (PM) or frequency modulation noise (FM), and is the attribute that characterizes the short-term frequency instabilities.

The two noise sources can be modeled in an equation by

$V(t)= Vp\ (1+\varepsilon(t))\ \sin(2\pi f_o t + \varphi(t))$ equation 3

Where ε(t) is introduced as the amplitude modulation noise, and φ(t) is the phase modulation noise. Both quantities are modeled as time dependent. Figure 3 shows the effects of amplitude noise modulation for the signal in figure 2. For this example $\varepsilon(t)=(t \times -2\times 10^5)-0.5$. ε(t) is typically an insignificant source of noise for voltage control crystal oscillators (VCXO) with square wave outputs. It will be ignored for the remainder of this application note.

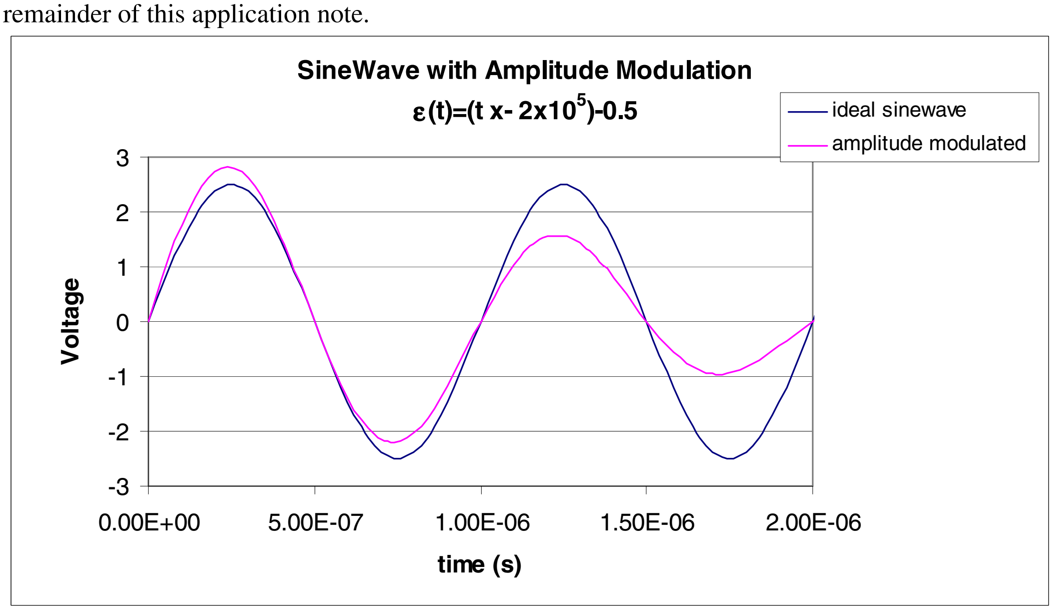


Figure 3: Amplitude modulated signal

Figure 4 shows the effects of phase modulation noise for the signal in figure 1. For this example $\varphi(t)=2\pi\sin(1.5\pi f_o t)/15$

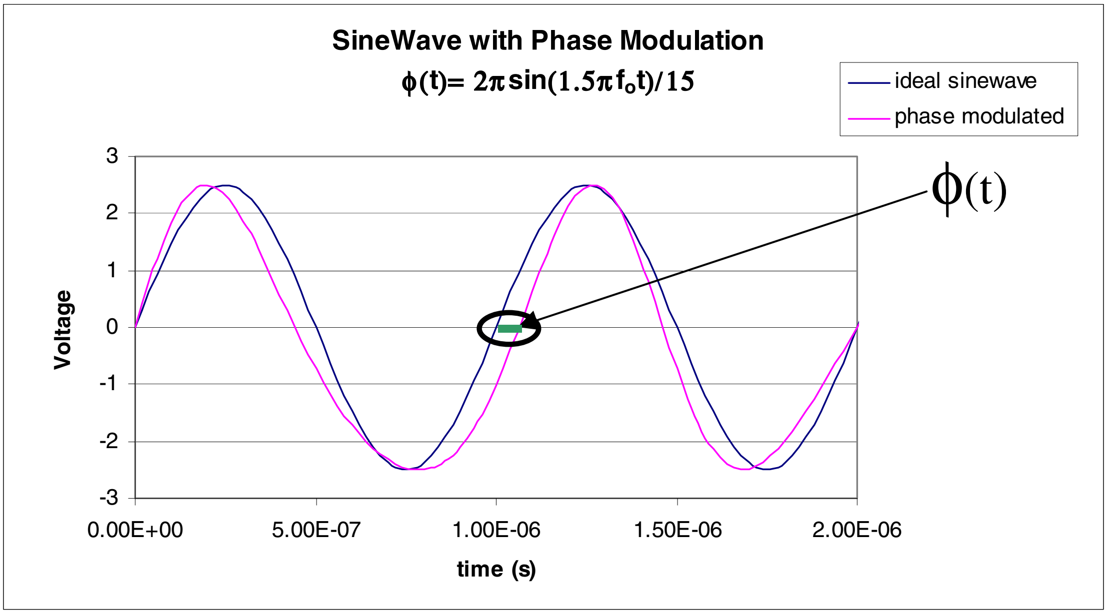


Figure 4: Phase Modulated Signal.

For square wave VCXO noise characterization, φ(t) is the dominant noise source, and typically of most concern for telecom and digital applications.

Units of measure for φ(t)

Seconds (s) – for the sample above the value of φ(t) is 80 ns measured at the end of the first cycle at the positive rising 0 Volt reference level.

Unit Intervals (UI) – The ratio of φ(t) to the period of oscillation. For this example it would be .08UI.

Radians (rad) – the radian value for φ(t) is in appropriate units to place in equation 1. To determine the radian value use the formula 2π x UI. For this example = .502 rad

Unlike the example shown in figure 4, φ(t) is typically random in nature, however different mechanisms of an oscillator will have specific effects on the spectral content of φ(t). These mechanisms are not discussed in this application note, but are covered in the application note "Frequency Stability in the Time Domain" by Mike Wacker avalaible on the Corning Frequency Control website at www.corningfrequency.com.

Telecommunications applications are typically concerned with the spectral content of φ(t) (expressed as the spectral density $S_\varphi(f)$) or some representation of it, which may be measured through a phase noise system. For a description of the spectral content of noise, see Appendix A at the end of this note. For many digital applications, with larger φ(t) tolerances (on the order of 10ps or greater), the time domain measurement and representation of φ(t) suffices.

The mechanics of accurate measurements, and the formulas involved can become very lengthy. This application note focuses on qualitative discussion of the methods, and graphical representations of the results.

Phase Jitter – Time Domain Analysis – Digital Jitter

Time Domain analysis of φ(t) is the easiest to measure, and the easiest to visualize. The time domain technique discussed in this application note is performed with a digital oscilloscope[1]. The Direct RAMBUS Clock Generator Validation Specification uses digital oscilloscopes to analyze jitter. This standard is widely used in the computer industry, however there is limited use for this method in the telecommunications industry as the bandwidth of φ(t) is not well defined. There are several different types of measurements that can be taken with this configuration. Brief descriptions of three measurements are given.

Time Interval Error (TIE). TIE is a measure of φ(t) that compares the jittery clock to an ideal clock source operating at the long-term average frequency of the signal. To measure TIE, as well as any other time domain method, a reference level and edge of the waveform must be specified. TIE is measured by subtracting the time of crossing through the reference level by the clock from the ideal crossing location. If a reference level is not defined, 0 volt rising edge (for an AC coupled signal) is assumed. This is shown in figure 5.

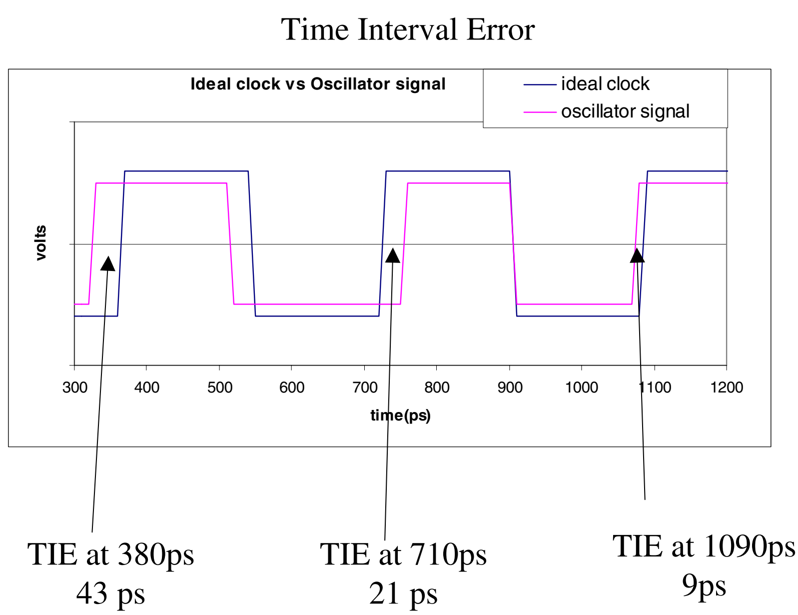


Figure 5: Three measurements of Time Interval Error (TIE).

Cycle-to-Cycle Jitter. Cycle-to-cycle jitter compares the difference in the period length of adjacent cycles. This would be calculated by subtracting period τ_1 from period τ_2 in the example shown in figure 6. Again the reference level and edge to measure must be specified.

Period Jitter. Period jitter compares the length of each period to the average (τ_{ave}) period of an ideal clock at the long-term average frequency of the signal. Each datapoint would be generated by subtracting τ_n-τ_{ave} where n is the period being measured.

[1] Contact David Chandler at dchandler@ofc.com for test equipment used in the report.

Cycle to Cycle and Period Jitter

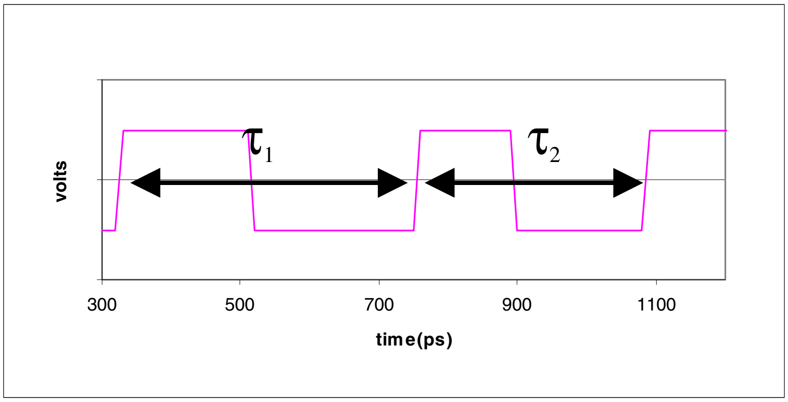
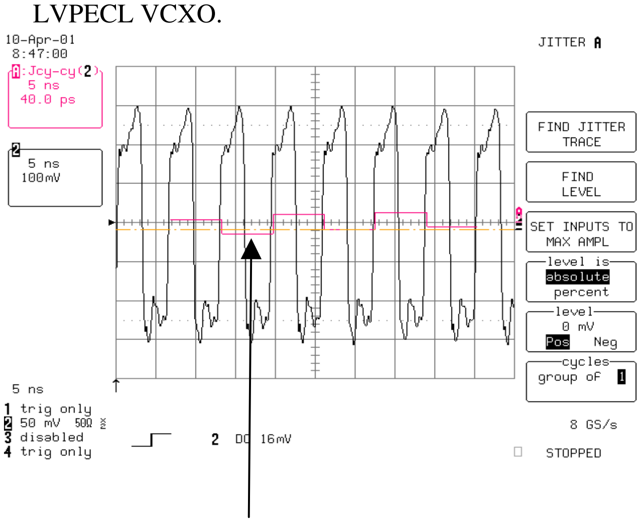
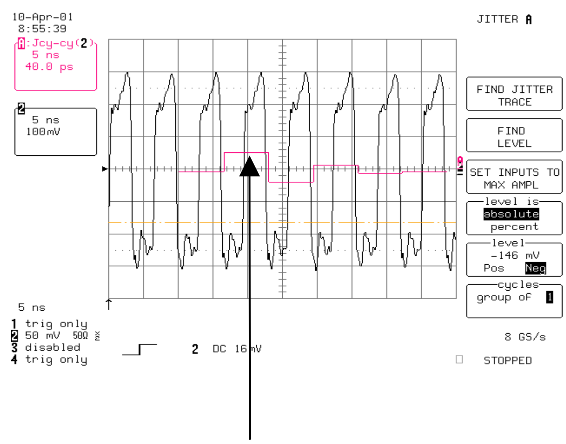


Figure 6: Adjacent cycles used to calculate cycle-to-cycle jitter.

When taking the three measurements, the reference level for the reading will have an impact on the results. The reference level specifies the point of each waveform to compare. Typically it is specified by the voltage level along the rising or falling edge. Figure 7 shows the difference in values obtained by changing the reference level from 0 volts positive edge to –145mV negative edge on a 155.52MHz, 3.3 Volts, LVPECL VCXO.



Cycle-to-Cycle Jitter
Reference Level -0 volts, positive edge
2nd readings is -5ps



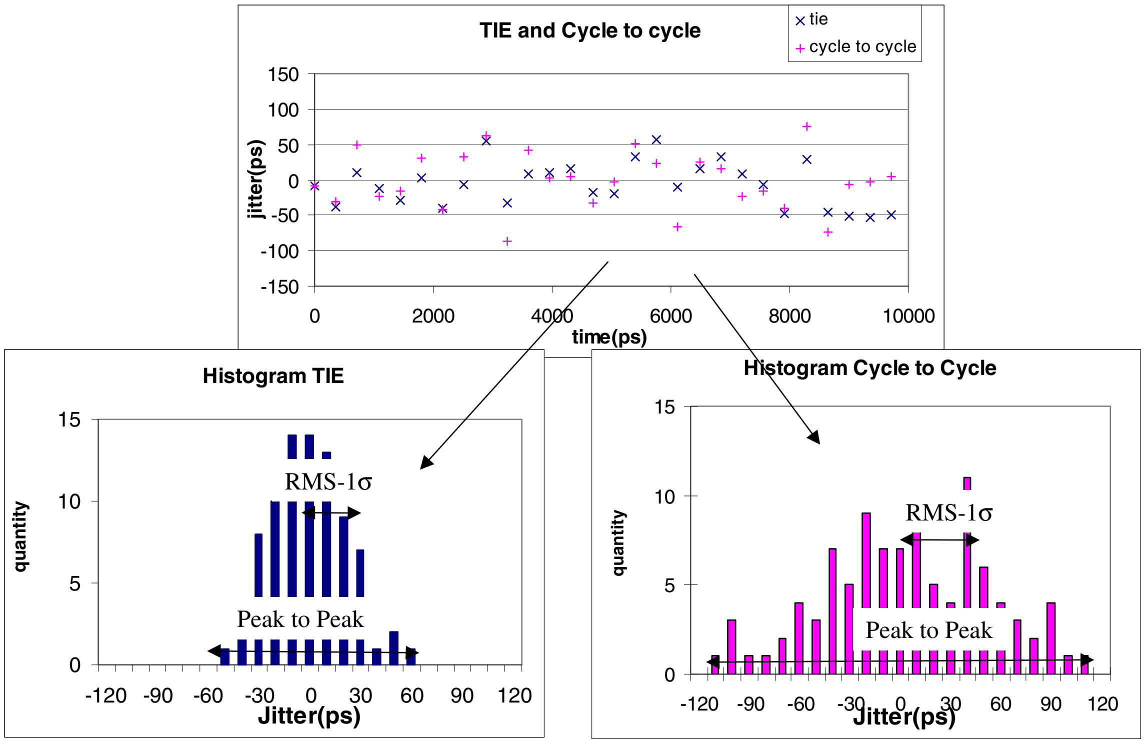
Cycle-to-Cycle Jitter
Reference Level --146 mvolts, negative edge
2nd readings is +24ps

Figure 7a: Red trace is cycle-to-cycle jitter.

Figure 7b: Red trace is cycle to cycle jitter

The two traces show a 29 ps difference in value when the jitter reading was taken at a different point on the waveform.

To provide descriptive statistics for all the above readings (TIE, Cycle-to-Cycle, Period) data sets for several adjacent cycles are collected. The peak-to-peak value and standard deviation (1σ), or root mean square (rms), are then calculated from these data points (for sinusoidal signals the rms value and standard deviation are approximately equal). Figure 8 shows data for Cycle-to-Cycle and Time Interval Error.



TIE	Cycle to Cycle	
99	99	number of hits
-58	-111	min
60	101	max
35.15587	49.11823606	stdev-RMS
118	212	pk to pk

Figure 8: Several adjacent data points are taken and placed in a histogram. The peak-to-peak values and standard deviation are used for the descriptive statistics of the population.

To obtain meaningful results using a digital oscilloscope a system should have the following capabilities:

1. High sampling rate (8 Gigasamples/second or greater)
2. Good post processing software.
3. Large single shot memory (10 Mega points or greater)
4. Low noise internal clock for the sampling rate.

Appendix C discusses these requirements in more detail. With these requirements met, repeatable values down to 4ps rms (+/-1ps cycle-to-cycle) are obtainable using Digital - Time Domain techniques. Manufacturers of digital oscilloscopes advertise rms noise floors using these methods as low as 2ps. While this method has a floor that is relatively high for the measurement of VCXOs, the contribution to the measured φ(t) may be calculated if the jitter due to the oscilloscope and the remainder of the test set up are known. The measurement error of the equipment and the jitter of the VCXO contribute to the total $\varphi_{measuredrms}$ as

$$\varphi_{measuredrms} = (\varphi_{vcxorms}{}^2 + \varphi_{equipmentrms}{}^2)^{1/2} \qquad \text{equation 5.}$$

Where

$\varphi_{measuredrms}$ is the recorded value
$\varphi_{equipmentrms}$ is due to the oscilloscope and all other equipment related noise
$\varphi_{vcxorms}$ is the actual jitter due to the vcxo.

The graph in figure 9 is a plot of $\varphi_{vcxorms}$ vs. $\varphi_{measuredrms}$ for three different $\varphi_{equipmentrms}$ (2, 3 and 4ps). For an oscillator with a $\varphi_{measuredrms}$ value of 5 ps, the oscillator noise is comparable to the equipment noise, however when the measured value is 10ps, the oscillator is the major contributor to the noise reading.

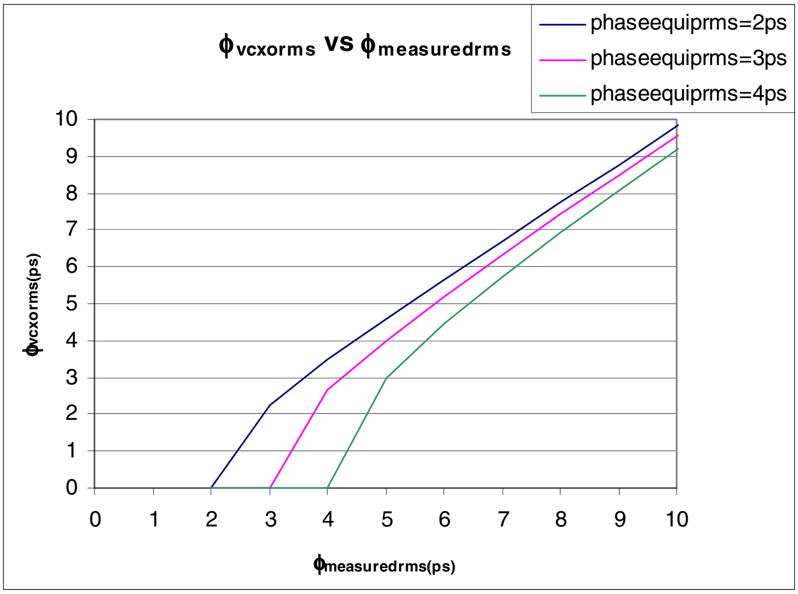


Figure 10: $\varphi_{measuredrms}$ vs $\varphi_{vcxorms}$ using a digital oscilloscope.

Frequency Domain Analysis Techniques – Phase Noise and Phase Jitter - Communications Jitter.

The spectral content of φ(t) is a concern in the communications industry, particularly when building SONET compliant devices. To obtain operating frequencies at several Gigahertz, the output of a square VCXO is multiplied through a phased-locked loop (PLL). The jitter is magnified by the PLL, thus placing tighter requirements on the oscillator than the final device. The spectral content is typically specified at discrete points (via phase noise), or as φ_{rms} of a specified bandwidth (phase noise integration – phase jitter rms).

Phase Noise

Phase noise is another measure of φ(t), but the results are displayed in the frequency domain as L(f). The displayed results are a comparison of the noise power at undesired frequencies to the total response power, normalized to a 1Hz bandwidth. For the oscillator in figure 11, the phase noise at 1kHz away from the carrier is the ratio of the red area (1Hz bandwidth at 1kHz) compared to the entire area under the curve.

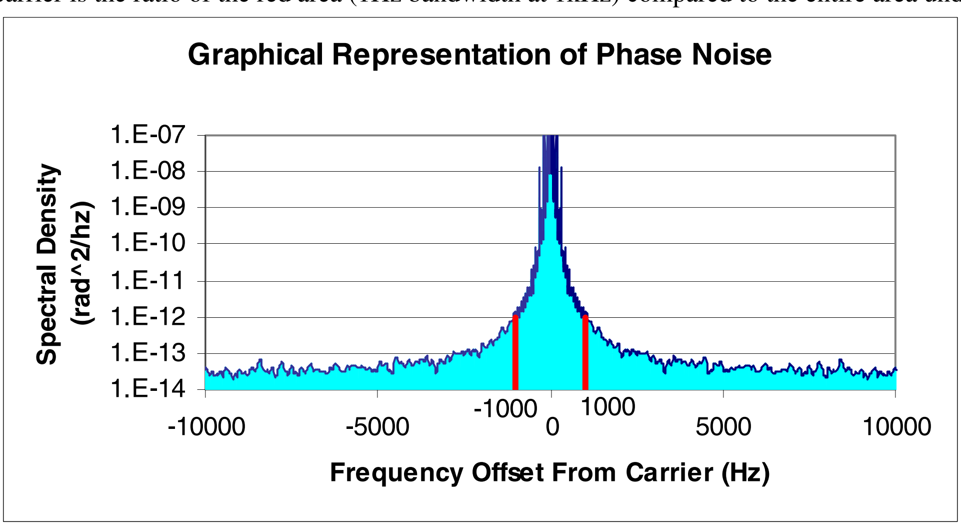


Figure 11: Graphical representation of phase noise. The phase noise at 1000Hz is the ratio of the red area to the blue area .

Commercially available phase noise test equipment measure φ(t) using the following phased-locked loop test set up.

Phase Noise Test System.

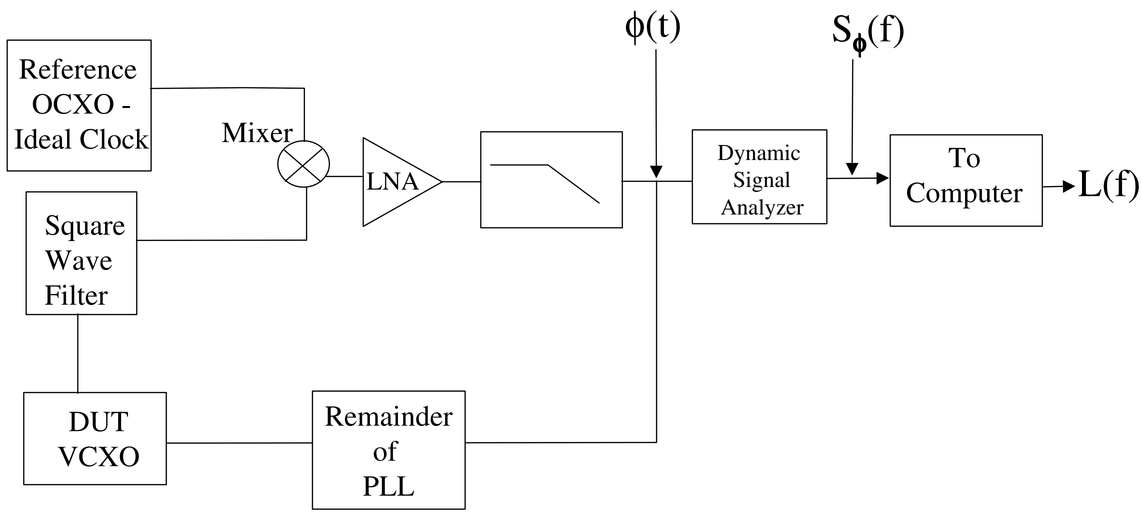


Figure 12: Standard Phase Noise Test Set Up.

For this test set up, the reference clock is an ovenized crystal oscillator (OCXO), which is approximately an ideal sinewave when compared to a VCXO, while the other is the VCXO (DUT). The frequency of the two devices must be close enough to allow an exact match in frequency with only minor changes to the control voltage of the VCXO.

By maintaining approximately a 90° phase shift between oscillators, the mixer output measures the small phase difference between the two signals. The signal coming off the mixer is sent through an amplifier and low pass filter. The signal that comes through the filter is a measure of φ(t). This is then measured on a dynamic signal analyzer (a high-resolution spectrum analyzer) which displays the spectral density of φ(t) in the frequency domain as $S_{\varphi}(f)$. Phase noise equipment takes advantage of the relationship

$S\varphi(f) = 2L(f)$ *for* $\varphi << 1$ *radian* equation 4

to plot phase noise. The base 10 logarithm of this value is then plotted in dBc/Hz.
Figure 12 is a phase noise plot for 155.52 MHz, 3.3Volt LV PECL VCXO.

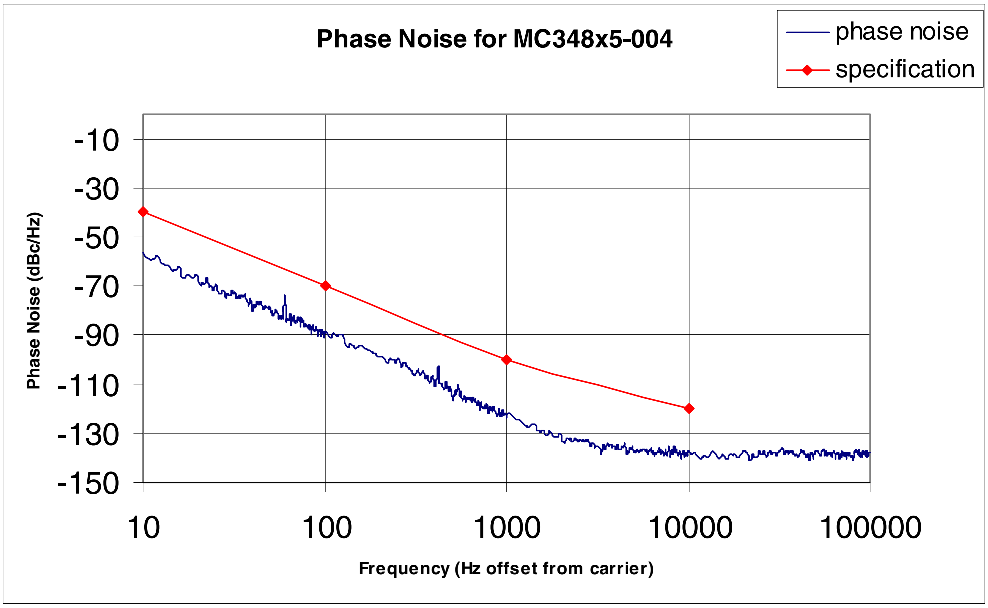


Figure 12: Phase Noise Plot for a 155.52Mhz, 3.3V, LVPECL VCXO.

Noise floors of commercially available phase noise equipment are as low as –170dBc/Hz.

For several communications applications, only the noise at certain frequencies away from the carrier is a concern, so the phase noise is specified at these frequencies.

Example of Phase Noise Specification.

Hz from Carrier	L(f) in dBc/Hz
10	-40
100	-70
1000	-100
10000	-120

This specification is plotted on figure 12.

Phase Jitter – Phase Noise Integration.
For many communications applications, the total noise power over a specific range of frequencies – not the shape (as shown in figure 12) is the primary concern. In order to determine the power over the frequency range (bandwidth), the time domain signal must be analyzed in the frequency domain, and then reconstructed in the time domain into an rms value with the unwanted frequencies excluded. This may be done by converting L(f) back to $S_\varphi(f)$ over the bandwidth in question, integrating and performing some additional calculations. The result is an rms value for $\varphi(t)$. The result may be expressed in dB, radians, unit intervals, or seconds. The value, in seconds, represents 1 standard deviation of phase jitter contributed by the noise in the defined bandwidth. Figure 13 graphically displays the integrated area in green. The bandwidth for the integration in this example is 500 to 10000Hz..

Limits of Integration

min frequency	500	Hz
max frequency	10000	Hz

total area under integral	phi rms (radians)	trms (s)
2.63264E-09	5.13093E-05	5.25085E-14

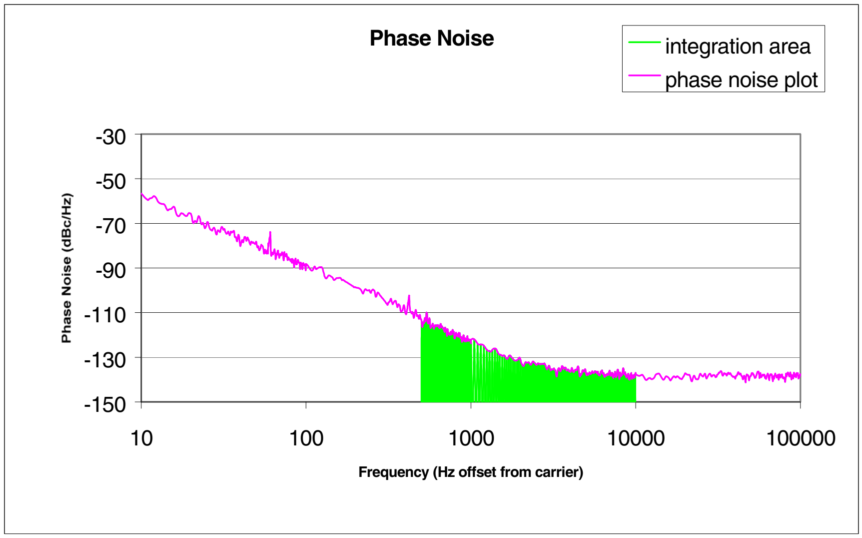


Figure 13: Phase Noise Integration to determine bandwidth limited (500 to 10000 Hz) phase jitter

Note that the bandwidth can have a significant impact on φ_{rms}. Figure 14 shows the integration on the same oscillator with a bandwidth of 10 to 10000 Hz.

Limits of Integration

min frequency	10	Hz
max frequency	10000	Hz

total area under integral	phi rms (radians)	trms (s)
1.70512E-05	0.004129311	4.22582E-12

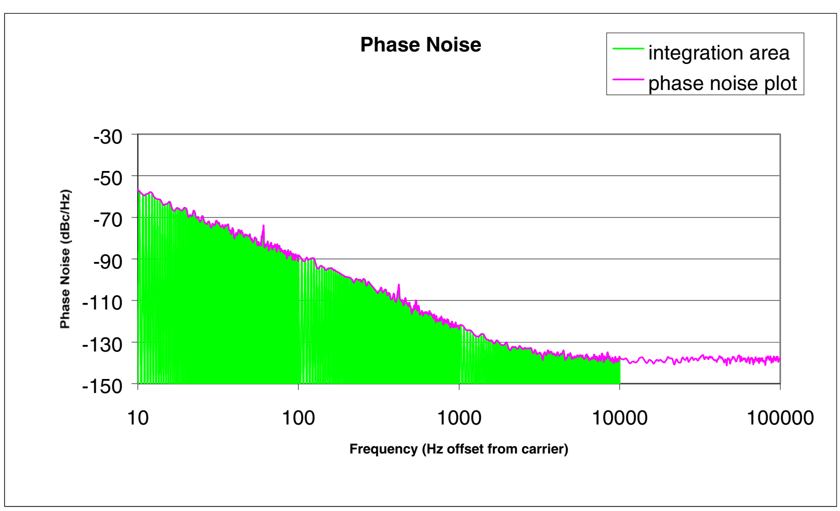


Figure 14: Same plot as figure 13 – bandwidth changed to 10 to 10000 Hz. Note that trms is ~ 80 times larger over this bandwidth.

Several telecommunications standards differentiate between jitter above and below 10Hz, calling the jitter below 10 Hz wander.

The trace integration can be performed by commercially available phase noise equipment, or by exporting the data from a phase noise machine into a spreadsheet program.

While this process seems cumbersome, it is one of the most effective ways to accurately provide an rms value for φ(t) over a specified bandwidth. Figure 15 summarizes the process used to obtain the bandwidth limited phase jitter number.

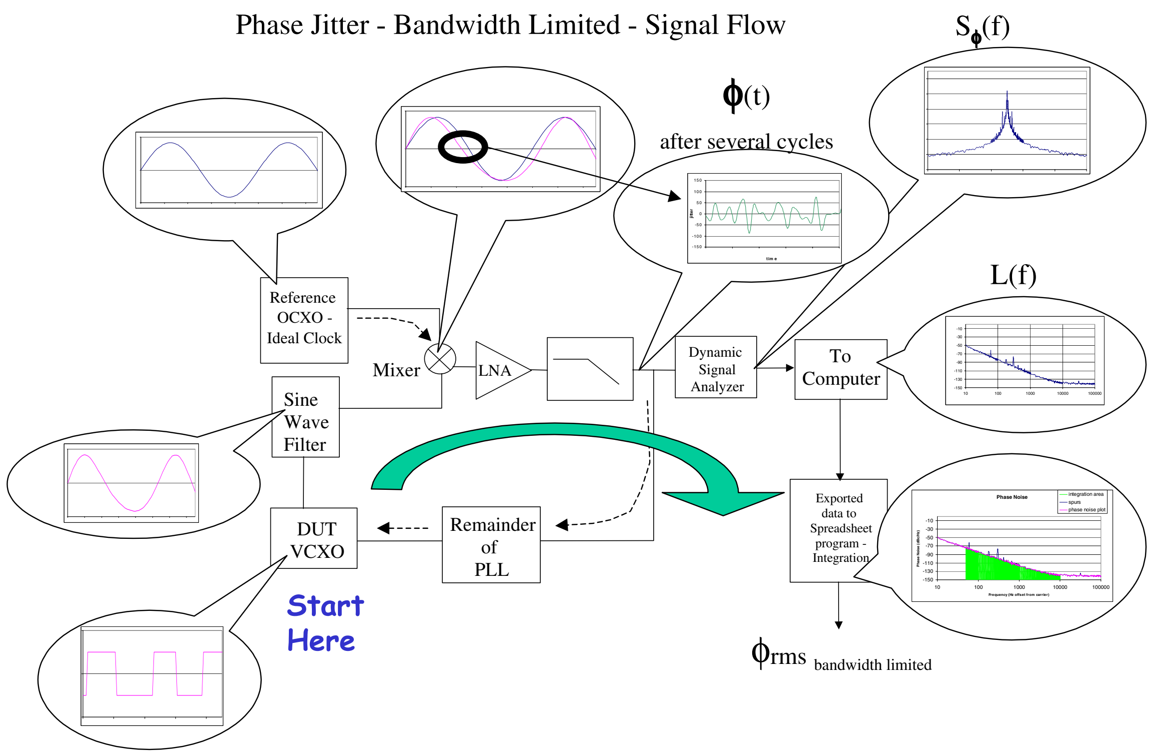


Figure 15: The process of obtaining a bandwidth limited phase jitter number.

Only φ_{rms} can be determined in this manner. The peak-to-peak value must be approximated. Appendix B discusses approximations and their validity.

Comparison of Test Methods

The following are representative of a 3.3Volt, 155.52Mhz, LVPECL VCXO.

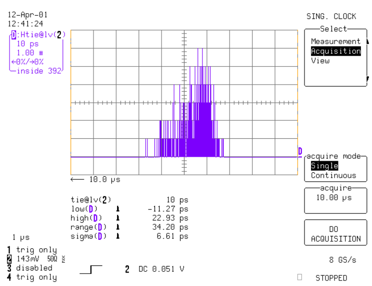


Figure16a: Time Interval Error - Time Domain
RMS: 6.61 ps

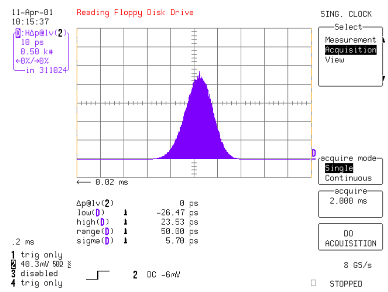


Figure 16b: Cycle to Cycle jitter - Time Domain
RMS: 5.70ps

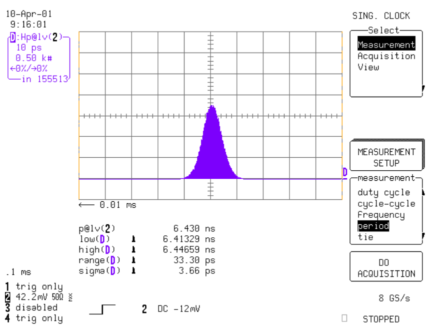


Figure 16c: Period Jitter -Time Domain
RMS: 3.66ps

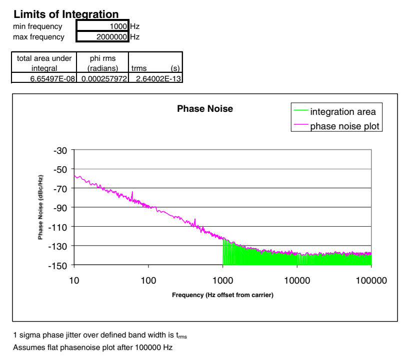


Figure 16d: Phase Jitter 1KHz - 2MHz - Frequency Domain
RMS: 0.264ps

Note the order of magnitude difference between the integrated phase noise plot and the time domain analysis. As explained earlier there are at least two major causes for these differences:

1) Definition of the bandwidth. The bandwidth is well defined and limited when performing a phase noise integration. There is no limitation of bandwidth on the oscilloscope method other than the equipment limitations.
2) The digital oscilloscope noise floor in figures 16a-16c is 2ps rms, and has not been removed from the numbers.

Specifying Requirements

Figure 16 shows the numbers vary greatly depending on measurement techniques. For this reason it is critical for a designer to understand the jitter tolerance for a VCXO in a higher level system. Different circuit design in a VCXO can yield markedly different jitter performance. Incorrect specification of the jitter can either cause system failures, if the specification inaccurately reflects system requirements, or can result in considerable additional cost because the jitter is overspecified and a more expensive VCXO is required to meet these specifications. The following minimum information should be provided when specifying jitter requirements.

Time Domain		Frequency Domain	
Cycle to Cycle		**Bandwidth**	
Period		>1kHz	
TIE		12kHz to 20MHz	
other		other	
Minimum Number of Cycles			
1000			
10000			
100000			
Reference Level			
Voltage			
Rising or Falling Edge			

Units of Measure	
Seconds	
Unit Intervals	
Radians	

Descriptive Statistics	
RMS - Standard Deviation	
Peak to Peak	

Bibliography

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2. Quartz Crystal Resonators and Oscillators for Frequency Control and Timing Applications, John Vig, 1997.
3. Introductory Electronics for Scientists and Engineers, Robert Simpson, 1987
4. Juran's Quality Control Handbook, Joseph Juran, 1988.

Appendix A: Spectral Content, Bandwidth Limitations, and RMS Values.

What does bandwidth limited spectral content of the phase noise actually mean? How does the RMS value represent the entire time dependent value for $\varphi(t)$?

Spectral Content.

All time varying signals may be represented as the sum of several sine waves of different magnitudes and phases. Figure A1 shows a non sinusoidal signal.

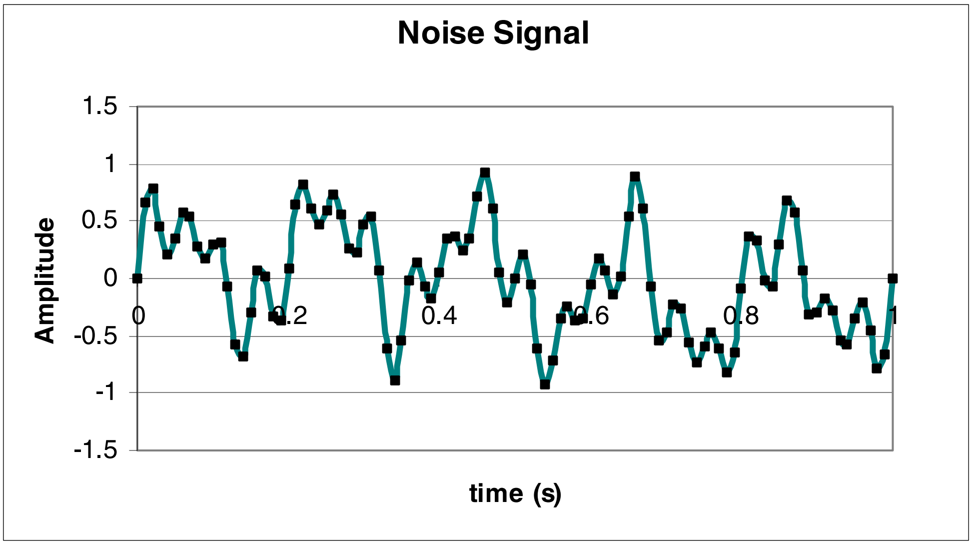

Figure A1: Non sinusoidal time varying signal.

This signal may be broken down into the following sine waves shown in figure A2. The frequency of these sine waves is listed in the legend, and the peak amplitudes may be determined by reviewing the y-axis.

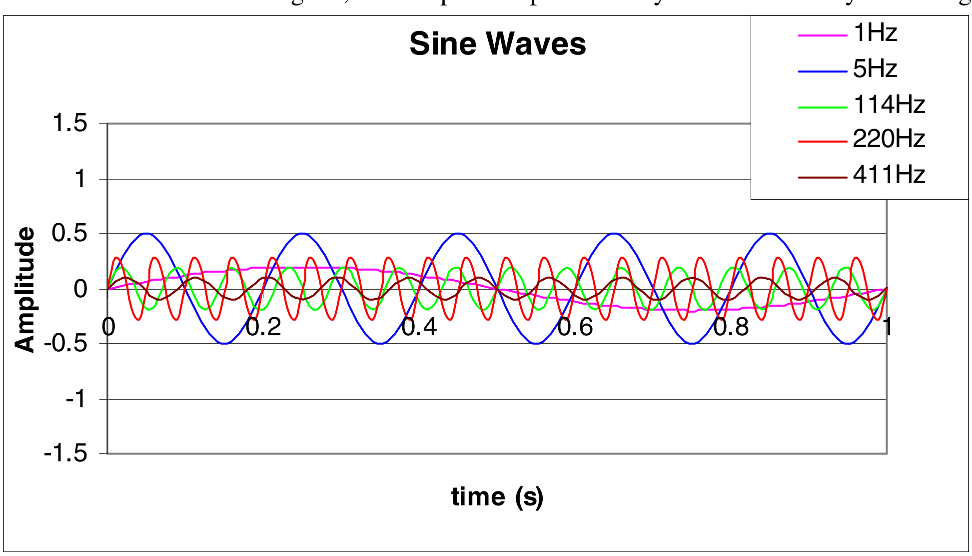

Figure A2: Sinusoidal Components of signal.

To see that these actually combine to form the signal listed above, simply add the amplitudes at any given point in time back together. The result of the summation, superimposed over the sinewaves, is shown in figure A3. Notice the summation is an exact replica of the original signal.

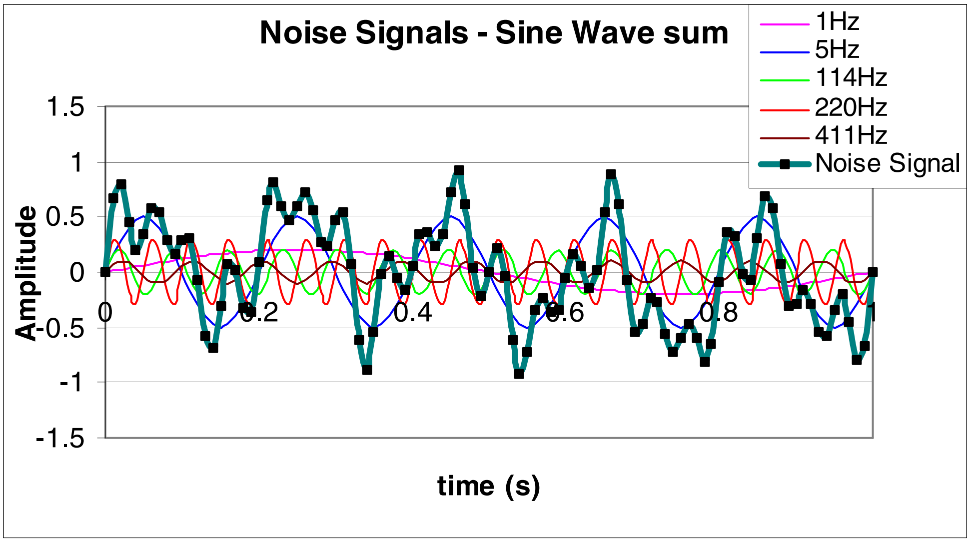

Figure A3: Sinewave reconstructed.

The peak magnitude of the sinewaves at the different frequencies is known as the amplitude spectrum of the signal, and can be viewed through a spectrum analyzer. Figure A4 is the spectrum for this signal:

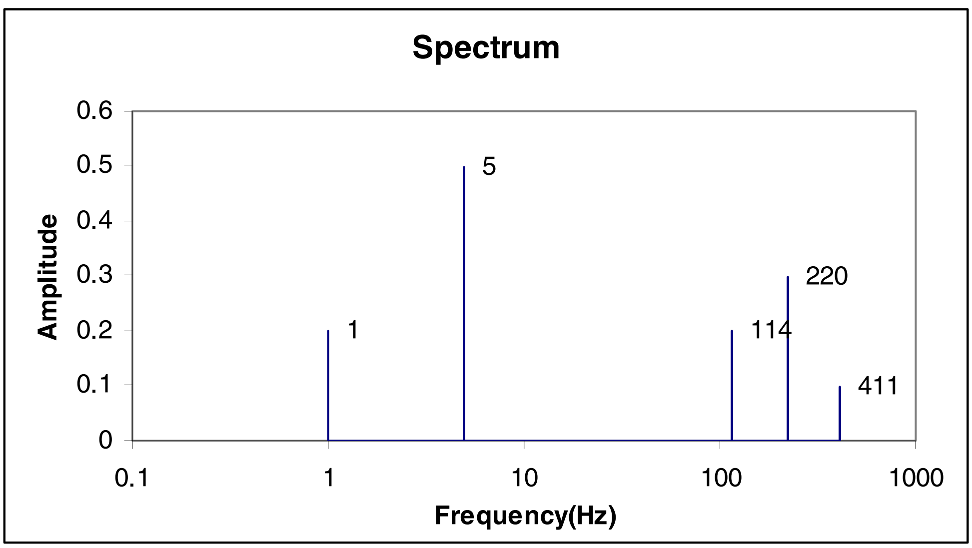

Figure A4: Amplitude spectrum for the signal in figure A.1

(Note: there is actually a phase angle for each sine wave as well. For this example the phase angle is zero for all frequencies). This view of the signal is referred to as the frequency domain. The process of determining which frequency creates the signal is known as Fourier Analysis and Fourier transformation. If the above signal was called $\varphi(t)$, then the following table represents $\varphi(f)$,

f	ϕ(f)
0<f<1	0
1	0.2
1<f<5	0
5	0.5
5<f<114	0
114	0.2
114<f<220	0
220	0.3
220<f<411	0
411	0.1
f>411	0

A signal that is non-periodic (does not repeat in time), has a continuous spectrum (a value for all frequencies), as opposed to the discrete spectrum shown in figure A4 (components only at distinct frequencies). For a random noise signal, rather than plotting φ(f) the spectral density S_φ (f) is plotted. S_φ(f) for a MC348x5-004 at 155.52 MHz are shown in figure A5.

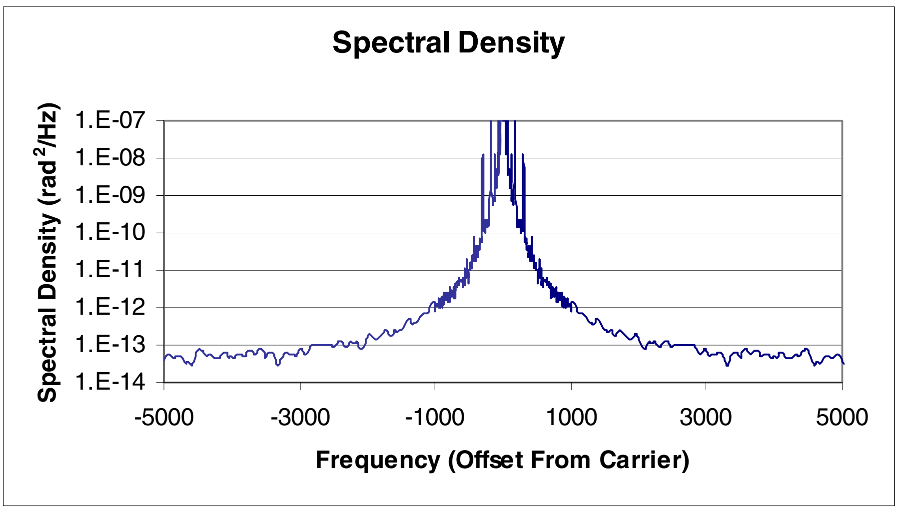

Figure A5: Spectral Density Plot for MC348X5-004

Bandwidth Limitations:
Several communications systems are only susceptible to noise within a given bandwidth. If a sine wave frequency component of the noise is not within the bandwidth, it will not degrade the performance of the system. For the example in figure A1 above, if the bandwidth is 10 to 300Hz, the 1,5 and 411 Hz will be filtered from the noise signal. Figure A6 shows the resulting waveform with these three frequencies removed.

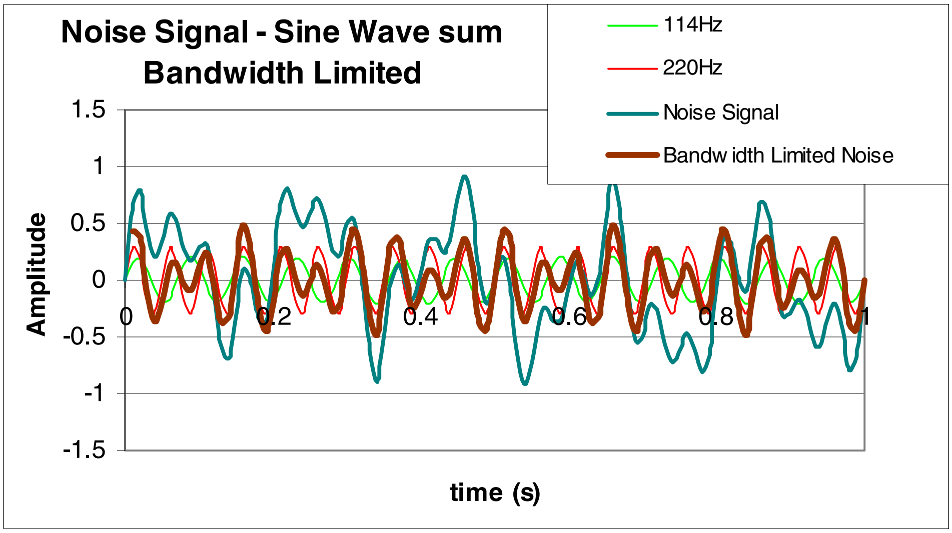

Figure A6: Bandwidth Limited signal

RMS Values

The average power dissipated by a sinusoidal signal is independent of the polarity (whether the sign is positive or negative). In order to calculate an effective time independent value for a signal that is independent of sign changes – the root mean square (rms) is calculated. The RMS formula for a function f(t) is

$f_{rms} = [(1/T)\int_0^T (f(t))^{2)}]^{1/2}$. equation A1

If two functions f_1 and f_2 are added to form a function $f_3(t)$

$f_3(t) = f_1(t) + f_2(t)$ equation A2

The value f_{3rms} may be calculated by either applying the formula for f_{rms} above, or, if the rms values for f_1 and f_2 are already known then

$f_{3rms}= (f_{1rms}{}^2+f_{2rms}{}^2)^{1/2}$ equation A3

For any sinusoidal signal the rms value is .707x peak amplitude.

For the signal φ(t) above, the bandwidth limited φ_{rms} from 10 to 300 Hz is .380, where as an unlimited bandwidth measurement of φ_{rms} would have given .655.

A similar summation method is used to convert a spectral density plot Sφ(f), into a bandwidth-limited φ_{rms}. The rms value is determined by obtaining the square root of the integral over the bandwidth:

$\varphi_{rms}=[\int_{f1}^{f2} S_\varphi(f))]^{1/2}$ equation A4

This is the method used to obtain φ_{rms} in figure 15.

Appendix B: Peak-to-Peak and Standard Deviation Values

The advent of digital oscilloscopes with large memory capabilities (able to store 100,000 cycles of data) has allowed more accurate descriptive statistics of φ(t) in the time domain. While this has added more validity to measurements, as well as repeatability, it has also caused specifications (particularly peak-to-peak, cycle-to-cycle measurements) to be redefined. The peak-to-peak values have grown as the sample sizes have increased, and the variation in the standard deviation has reduced significantly. For the purposes of discussion, φ(t) is assumed to be a normal distribution.

Effect of Sample size on Standard Deviation.

Due to a difference in confidence intervals, a digital oscilloscope that can only record 100 cycles will give a much less precise value for 1σ than an oscilloscope that can provide 1000 cycles.
The formula for a confidence limit on a standard deviation is

Upper Confidence Limit $=\sigma\ ((n-1)/(\chi^2_{\alpha/2}))$^$^{1/2}$ equation B1
Lower Confidence Limit $=\sigma\ ((n-1)/(\chi^2_{\alpha/2}))$^$^{1/2}$ equation B2

Where
σ is the calculated standard deviation
n is the total number of samples
1-α is the desired degree of confidence (chosen as 95%)
χ2 is the chi squared distribution with n-1 degrees of freedom at the appropriate α level.

The confidence interval is the range of values bounded by the upper and lower confidence limits. Figure B1 is a graph of true standard deviation vs. sample size for a measured value of 10ps.

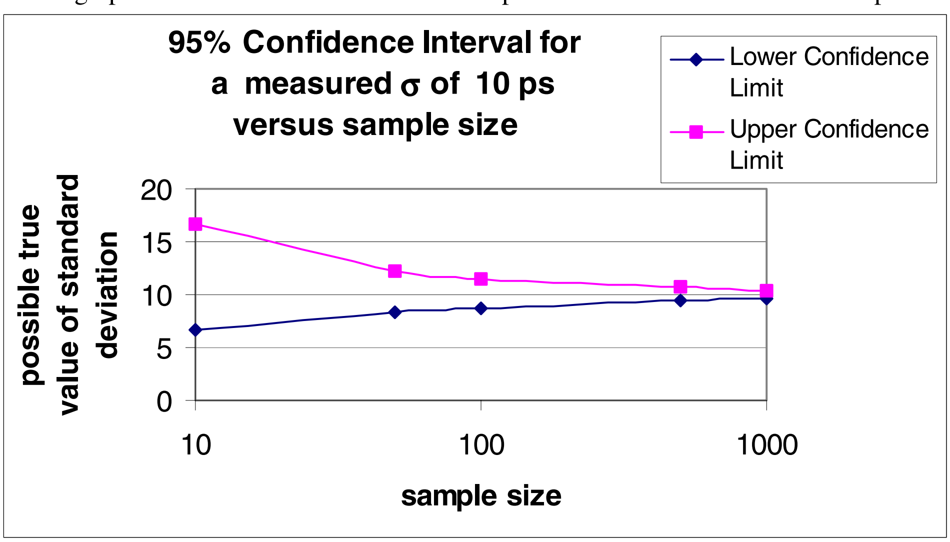

Figure B1: Confidence Interval versus sample size.

A 100 cycle population with a measured 1σ value of 10 would have a 95% confidence interval of 8.74 to 11.54, meaning there is a 95% probability the true value of 1σ is between these two values. A 1000 cycle population with a measured 1σ value of 10 would have a 95% confidence interval of 9.56 to 10.44. The net effect is that the ten-fold increase in sample size has created an improved confidence interval 3.18 times smaller.

Effect of Sample Size on Peak-to-Peak Values

Assume a jitter population for one oscillator is known to have a true 1σ value of 10ps and an average value of 0. Figure B.2 is the Gaussian (Normal Probability) distribution for this population.

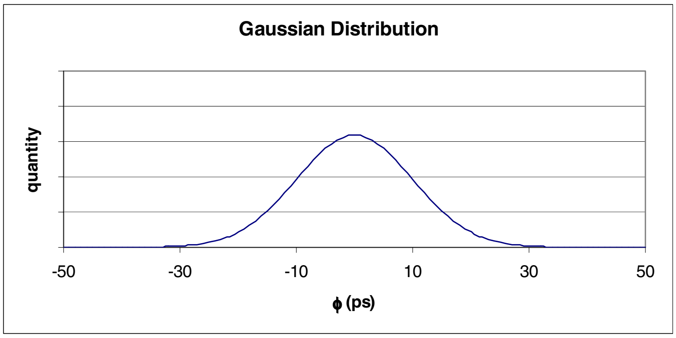

Figure B2: Gaussian distribution for a random process with a standard deviation of 10ps.

Figure B3 was generated using the binomial probability distribution and the normal probability distribution. It displays the minimum peak-to-peak value that will be detected (95% probability) given the number of cycles in the sample, assuming a standard deviation of 10ps.

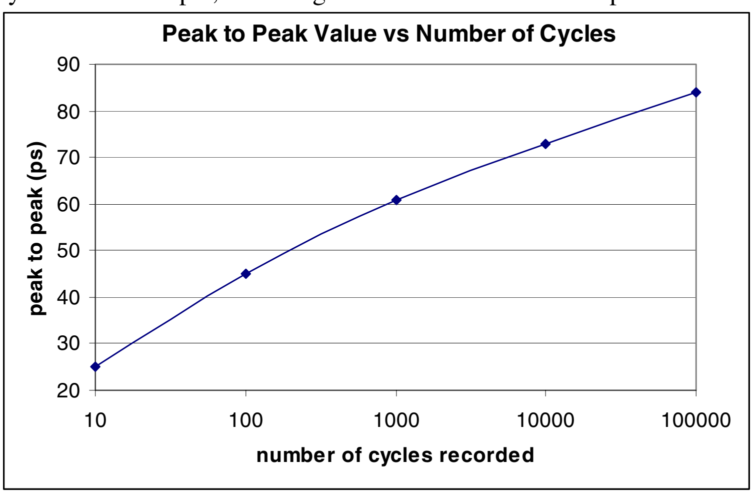

Figure B3: Minimum peak-to-peak value vs. number of cycle for 1σ=10ps (95%probability).

From this chart it can be seen that by increasing from 100 to 1000 cycles, the minimum peak-to-peak value increases from 45ps to 60ps. At 100,000 cycles the peak-to-peak value has increased to 84ps.

Approximation of Peak-to-Peak Values from the Standard Deviation.

When the phase noise integration method is used to obtain $\varphi_{rms,}$ or when the desired sample size for the peak to peak exceeds the capabilities of the oscilloscope, approximations for the peak-to-peak are made based off the standard deviation (or rms value).

The approximation method assumes the normal distribution with the standard deviation measured. There is a finite probability that any independent sample will be outside the approximated peak-to-peak value. This probability decreases as the number of standard deviations used to approximate the peak-to-peak value increases. Figure B4 plots the probability that any single independent sample will be outside of the approximation

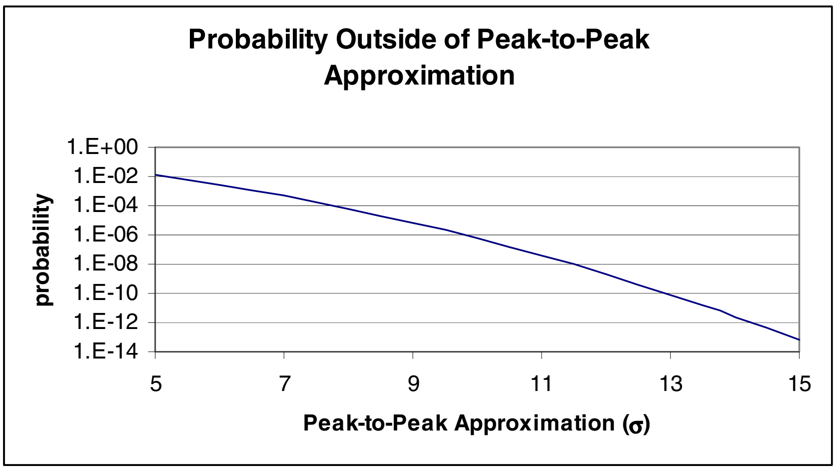

Figure B.4: Probability of sample outside of peak-to-peak approximation.

Appendix C: Digital Oscilloscope Requirements.

To obtain meaningful results using a digital oscilloscope a system should have the following capabilities:

1. High sampling rate (8 Gigasamples/second or greater)
2. Good post processing software
3. Large single shot memory (10 Megapoints or greater)
4. Low noise internal clock for the sampling rate.

Figure C1 displays the importance of the first two items. Good post processing software is needed when taking measurements with digital oscilloscopes. Each data point on the chart is a recording by the digital oscilloscope. Even at extremely high data sampling rates (8 Gigasamples per second in this case), there are still relatively few data points present on the edges. For meaningful time domain readings, the oscilloscope must also have good post processing software to interpolate the value at the reference level, because there is a high probability there will not be a data point at the exact location.

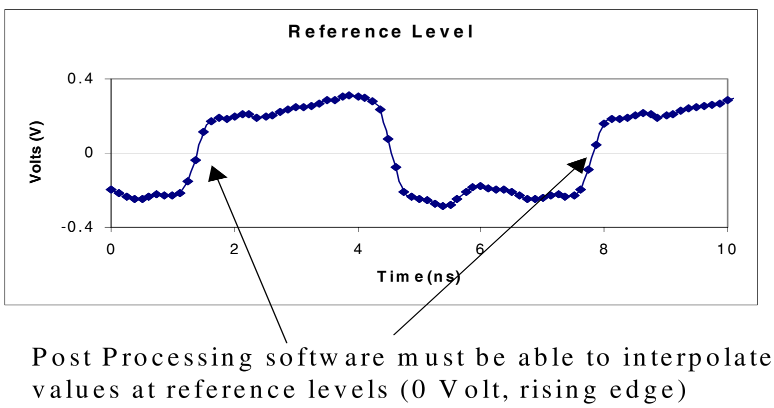

Figure C1: Data points recorded by digital oscilloscope. Note there are no recordings at the reference level.

Large single shot memory is another important requirement as it directly impacts the number of recorded cycles. The number of cycles obtained has an impact on the peak-to-peak value and standard deviation. Figure C2 shows the difference in the cycle-to-cycle peak-to-peak value for a sample of 100 cycles vs. 303,000 cycles. See Appendix B on peak-to-peak values, samples sizes, and Gaussian distributions for a more detailed explanation on the impact of sample sizes.

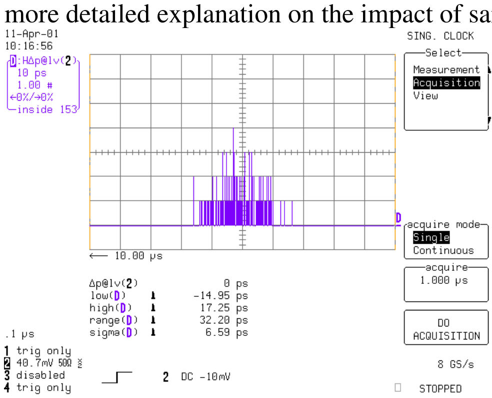

Figure C2a: Cycle-to-Cycle jitter 153 cycles. Peak-to-peak jitter 32.20ps, standard deviation (1σ) 6.59ps.

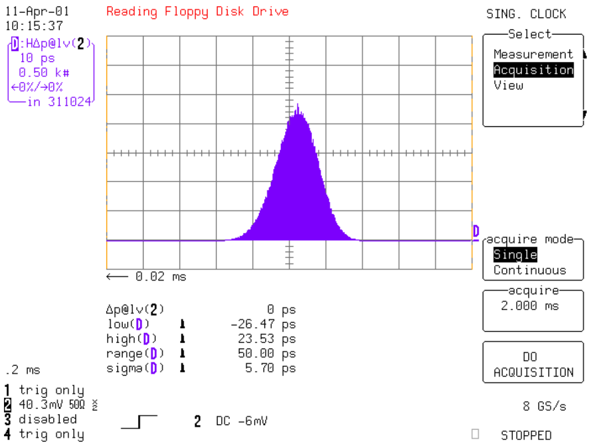

Figure C2b: Cycle-to-Cycle jitter 311,024 cycles. Peak-to-peak is 50ps, standard deviation (1σ) 5.79ps.

The final important factor is a low noise internal clock. The internal clock controls the sampling rate of the oscilloscope. Oscilloscopes that meet all of the requirements listed above advertise noise floors as low as 2ps rms.