DESIGN METHODS, TRANSISTOR MODELING, AND NUMERICAL SIMULATION OF LOW PHASE NOISE OSCILLATORS

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Must the design methods of low phase noise oscillators be revisited ?

- New design methods allowing to clarify design process, are regularly proposed

-New deep insights on phase noise generation in free running oscillators, are regularly presented

-New accurate models of transistors and their associated noise sources, are continually developed.

OUTLINE

- Introduction

- Design objectives for low-noise operation

- Modern linear design approach

- A simplified transistor model, valid for FETs and HBTs
- Open-loop gain concept: calculation of the starting oscillation frequency
- Fast analytic calculation of phase noise: Leeson formula revisited

- Nonlinear models and design tools

- Semi-conductor device modeling:
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- Conclusion

Design objectives for low-noise operation

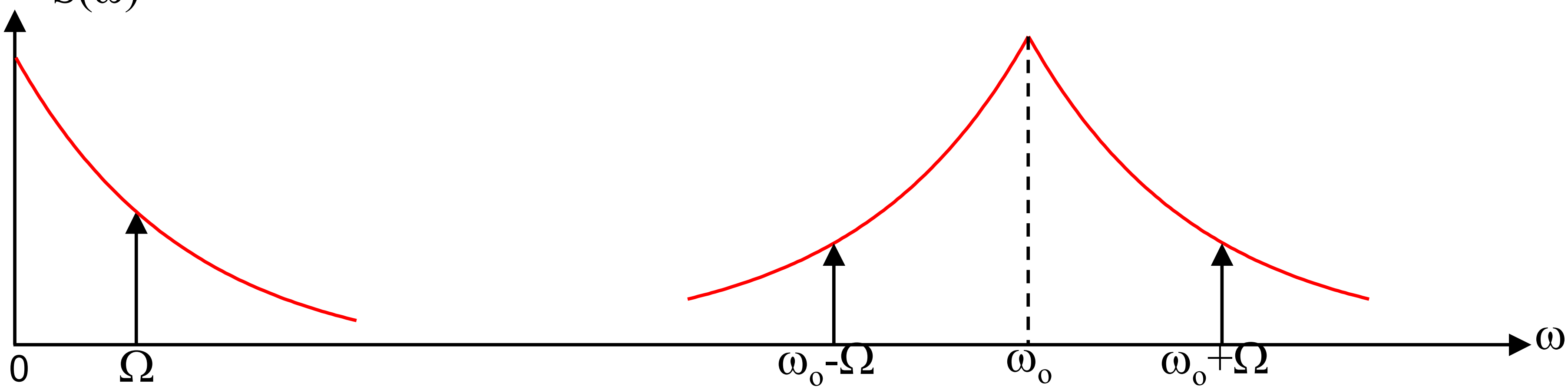


Fig.1 : Noise conversion in oscillators

A well-designed oscillator requires the optimization of the circuit behavior:

- on the one hand at the oscillation-frequency (large signal)
- on the other hand at the intermodulation frequencies resulting from the interaction between (small signal) noise sources and (large signal) steady-state.

Optimization objectives for low phase noise generation

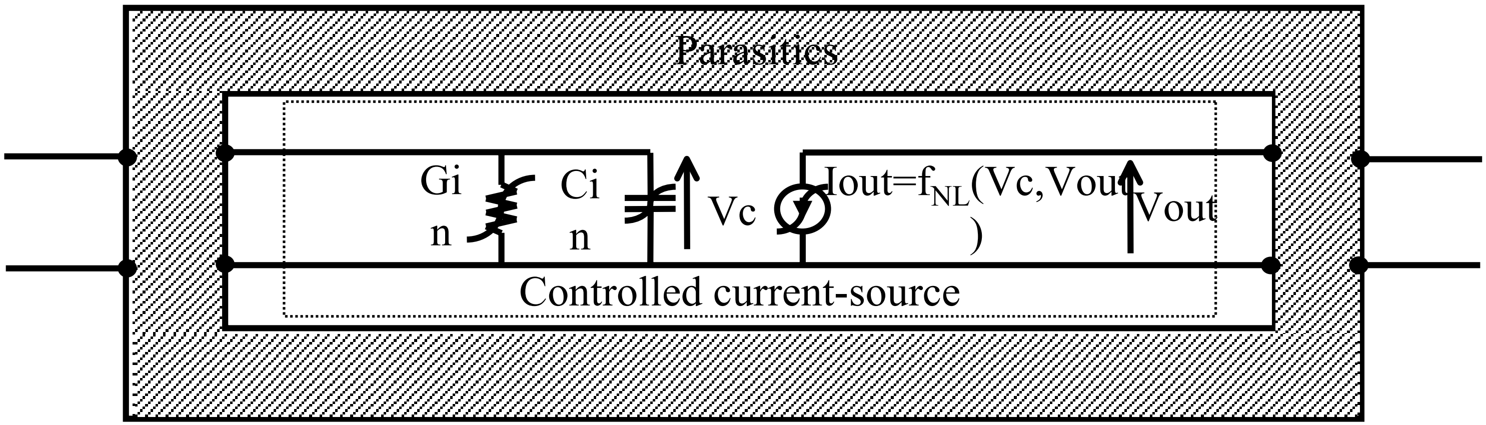
- At the oscillation frequency : ω_o
  - Maximization of the transistor added power
- Around the oscillation frequency :
  - Maximization of the phase slope of the oscillator open loop gain
- At low-frequencies near DC:
  - The impedance presented by the oscillator circuit and the bias network must be low in order to reduce the influence of the transistor low-frequency noise sources on the phase noise.

Modern linear design approach

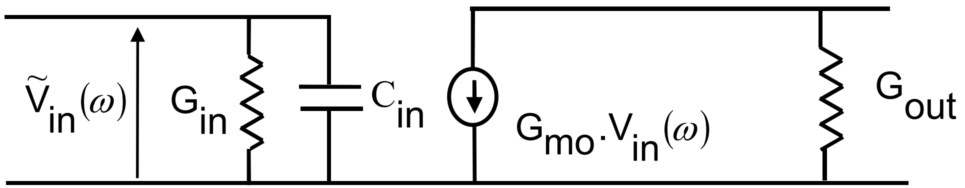
- A simplified transistor model:

From a circuit-design point of view, a transistor is fundamentally a non-linear voltage-controlled current source.

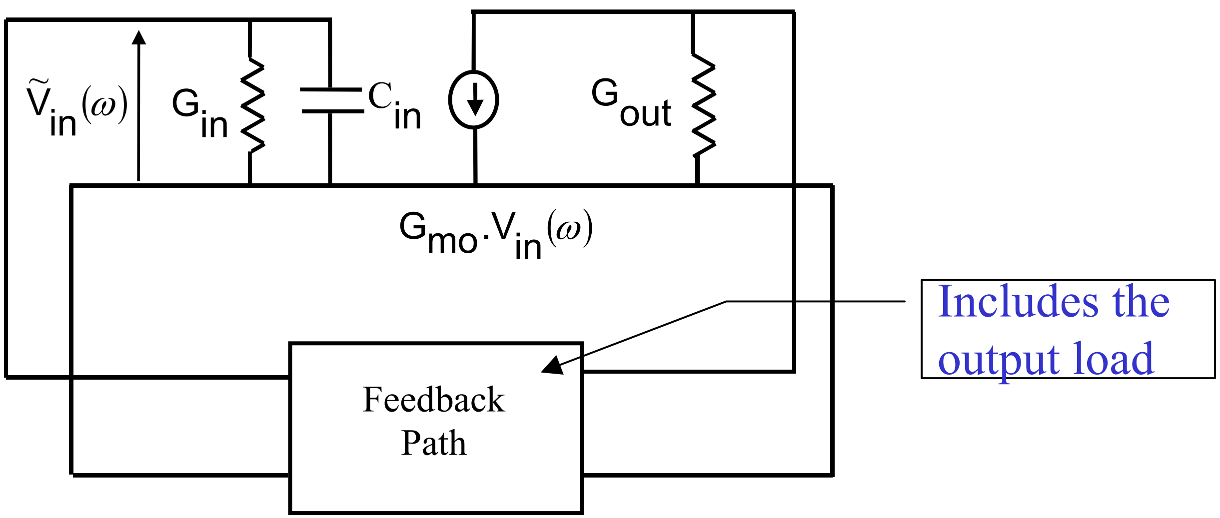
The controlling voltage is taken at a diode port : forward biased in HBTs and reverse biased in HEMTs. Due to the highly nonlinear equivalent capacitance of this input diode, the phase noise generation by up conversion takes place principally at this controlling voltage port.



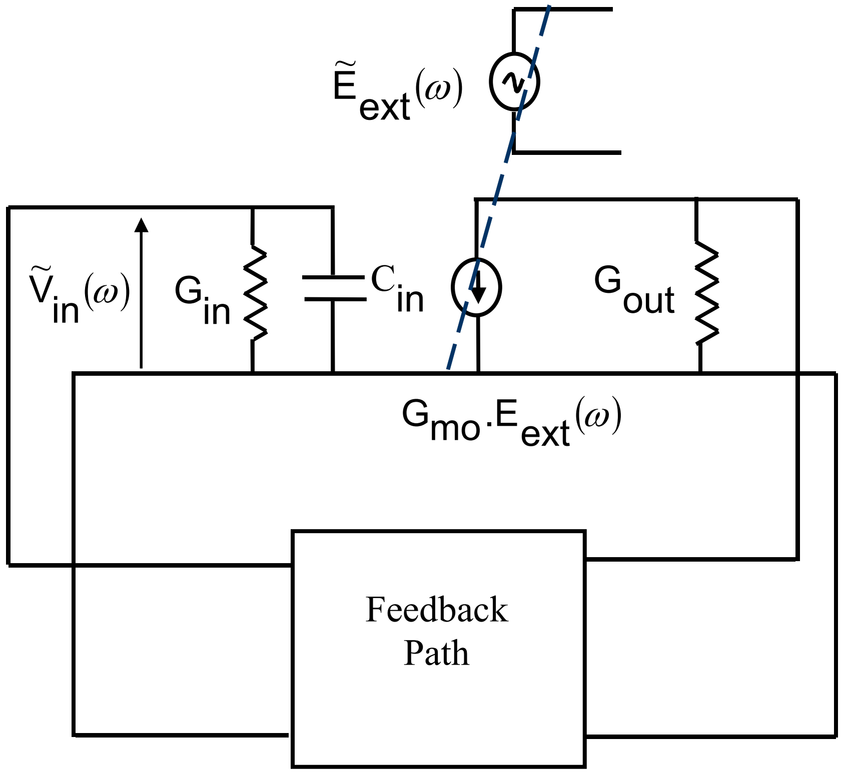
- A linearization of the nonlinear model gives the linear simplified model:



This model can be used for a first linear analysis of an oscillator circuit:



- The open loop gain concept:



$$\tilde{G}_{OL} = \text{open loop gain} = \frac{\tilde{V}_{in}(\omega)}{\tilde{E}_{ext}(\omega)}$$

- In a first small signal approximation, the oscillation frequency is given for the conditions

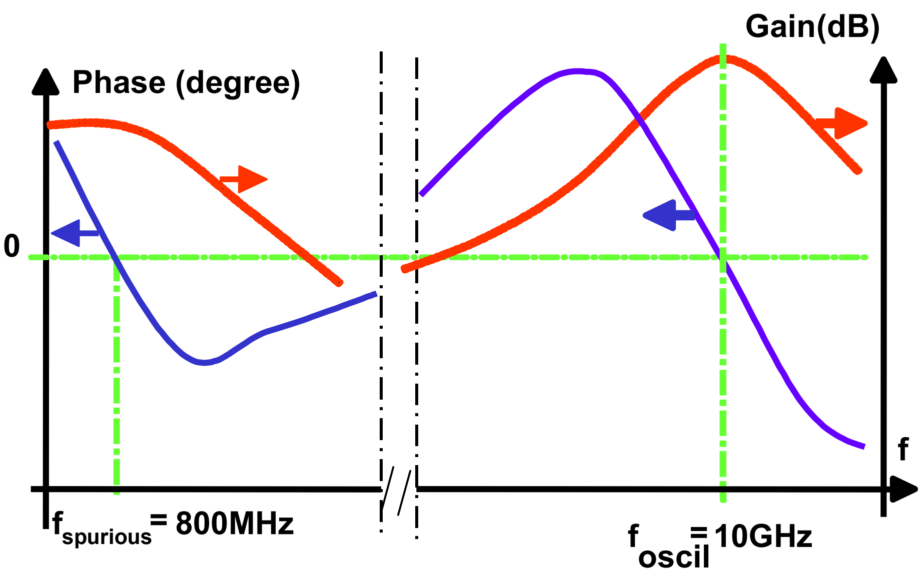
$$\begin{cases} \left|\tilde{G}_{OL}(\omega_O)\right| > 1 \\ \varphi_{OLG}(\omega_O) = \angle\tilde{G}_{OL}(\omega_O) \approx 0 \end{cases}$$

→ $\left|\tilde{G}_{OL}\right| > 1$: During the transient $\tilde{G}_{OL}$ reduces to a large signal operating point where

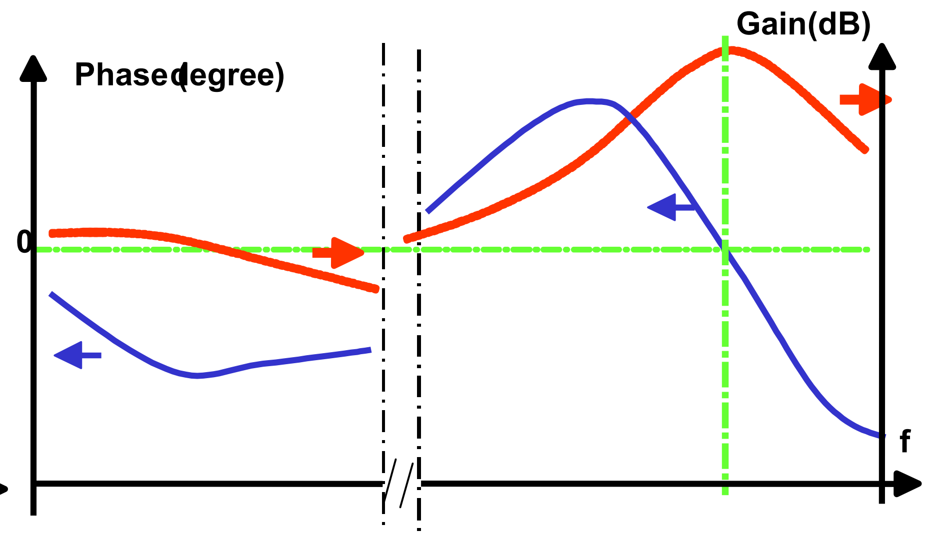
$$\left|\tilde{G}_{LS}(\omega_O)\right| = 1$$

→ $\varphi_{OLG}(\omega_O) = \angle\tilde{G}_{OL}(\omega_O) \approx 0$: During the transient $\varphi_{OLG}(\omega)$ shift in frequency to the operating point where $\varphi_{LS}(\omega_O) = 0$ and :

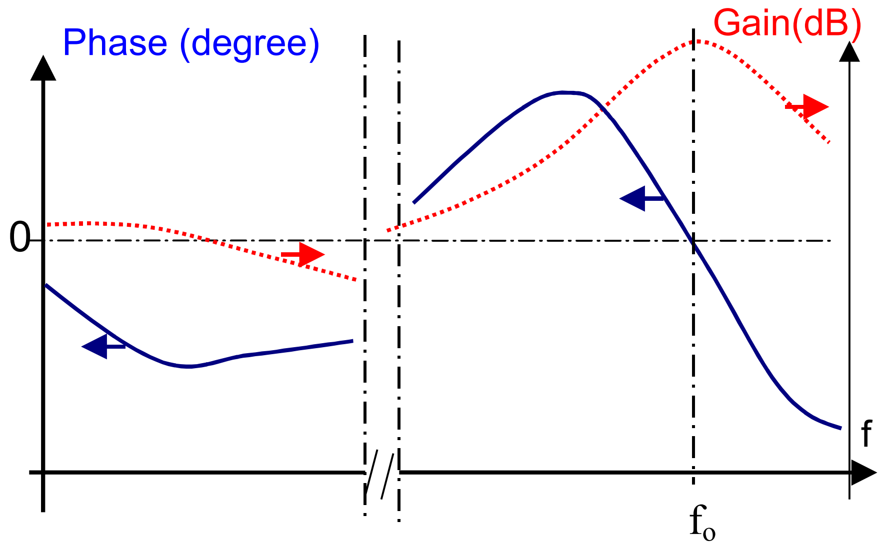
$$\omega_{O\,\text{large signal}} \approx \omega_{O\,\text{small signal}}$$



Simulation results of linear open-loop gain for an MMIC HEMT oscillator



Simulation results of linear open-loop gain for an MMIC HEMT oscillator with circuit stabilization



Simulation results of linear-loop gain for a transistor oscillator.

Question : Are the previous conditions sufficient to find a stable oscillation ?

Answer : No, the local stability of the large signal operating point must be analyzed.

→ See section “Non linear simulation tool”

→ A first response can be given by the linear analysis

If :

$$\begin{cases} \left|\tilde{G}_{OL}(\omega_o)\right| > 1 \\ \varphi_{OL}(\omega_o) = \angle \tilde{G}_{OL}(\omega_o) = 0 \\ \left.\dfrac{d\varphi_{OL}}{d\omega}\right|_{\omega_o} < 0 \end{cases}$$

Then the final steady-state oscillation should be stable

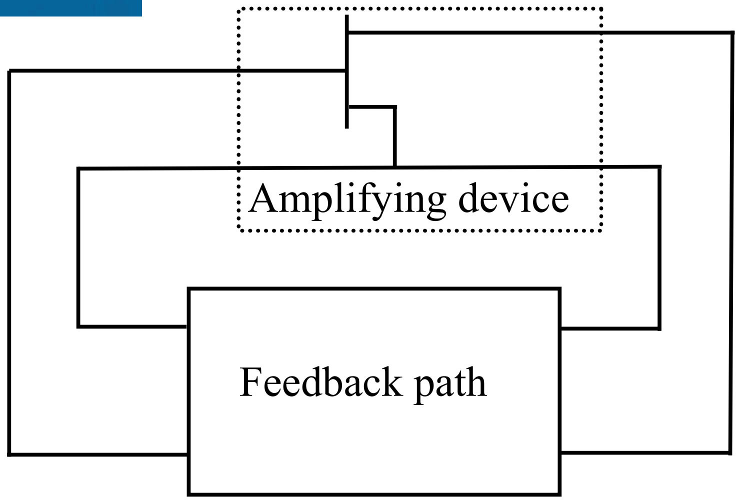
- Linear evaluation of phase noise in free running oscillators:

  - A useful tool: The modified Leeson formula

In the field of linear feedback-systems formalism, the Leeson formula is a useful tool for the determination of phase noise in feedback oscillators

A direct application of the Leeson model without care, can lead to erroneous results because the formula contains hidden parameters which are generally unwittingly ill-evaluated, or neglected.

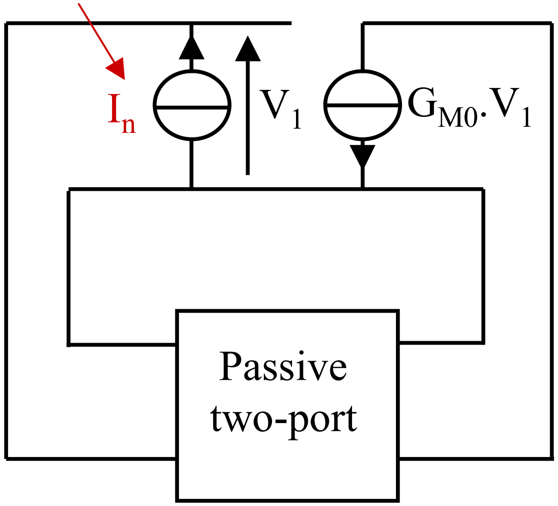
A detailed analysis enables us to enlighten the hidden parameters leading to a modified Leeson formula which is valid for all oscillator circuits.



This general representation of a feedback oscillator includes

- the amplifying device: the transistor
- the feedback path, which includes the load conductance

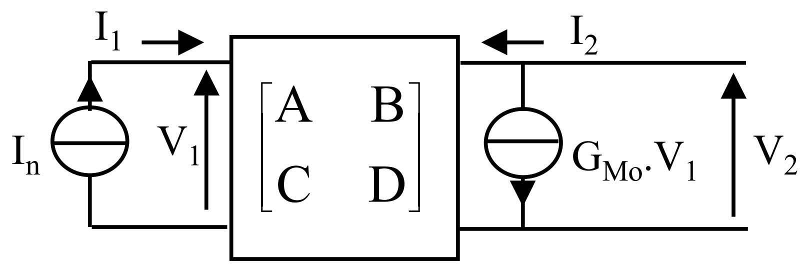
white noise source



A linear representation (after nonlinear equivalent linearization) of the feedback oscillator in the frequency domain allows to highlight:

The amplifying (active) function of the transistor :the voltage controlled current source of the transistor is isolated $G_{Mo}V_1$ from the passive elements of the transistor model, which are now included in the passive, reciprocal feedback path.

For phase noise calculation purposes a carrier voltage of peak value V_{o1} at the oscillation frequency ω_0 is implied at the controlling input port of the transistor.



Feedback oscillator with a chain matrix description of the passive circuit

One noise source alone is not always sufficient to characterize a noisy transistor.

However in the most general case two correlated sources :

- $In1_{input}$ and $In2_{input}$ are sufficient.

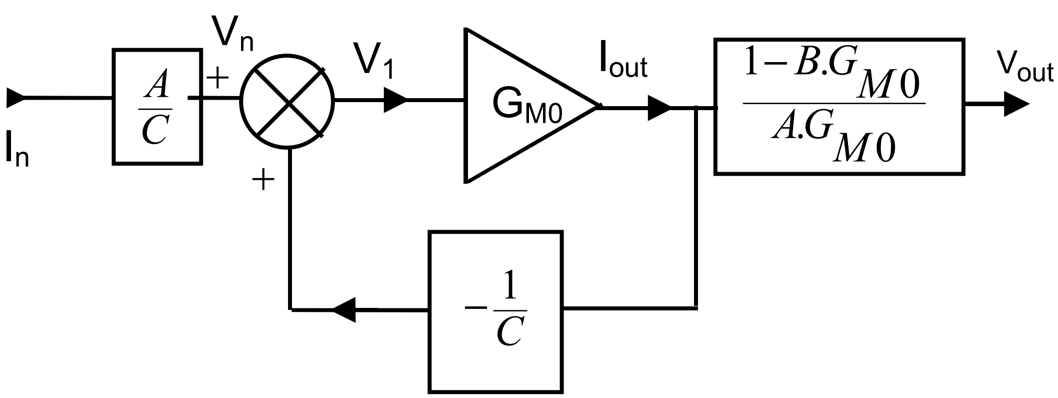
In that case $\langle |In|^2 \rangle$ becomes $\langle |In|^2 \rangle = \left\langle \left| In1_{input} \right|^2 \right\rangle + \left\langle \left| In2_{input} \right|^2 \right\rangle + 2\Re\left(In1^*_{input} In2_{input} \right)$

For phase noise calculation, this input noise source must be carefully evaluated according to the localization of the physical noise sources in the transistor

Determination of $S_{\Delta\varphi in}$

In the field of linear feedback system formalism, the equivalent closed loop representation of the figure below is deduced.
In this figure, we have: $G_{M0}.V_1 = I_{out}$, $V_2 = V_{out}$,
V_n represents the equivalent input noise voltage source due to I_n.

In order to determine $S_{\Delta\varphi in}$, we first calculate V_n around the oscillation frequency ω_o.



Let $\omega_n = \omega_o + \Delta\omega$

We obtain

$$\left\langle \left| V_n \right|^2 \right\rangle = \left\langle \left| I_n \right|^2 \right\rangle \left| \frac{A_0}{C_0} \right|^2$$

A_o and C_o are the chain-matrix coefficients taken at ω_o

Determination of $S_{\Delta\varphi in}$ (continued)

A conventional treatment shows that by addition to a pure carrier signal of peak value V_{o1} at the frequency ω_o, two uncorrelated components at $\omega_o+\Delta\omega$ and $\omega_o-\Delta\omega$ give rise to a modulated carrier with a phase-noise spectral density $S_{\Delta\varphi in}$

At an offset frequency $\Delta\omega$ from the carrier, we obtain

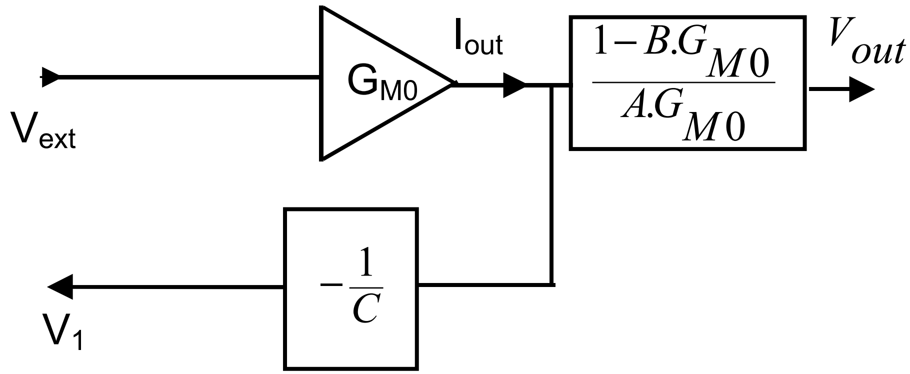
$$S_{\Delta\varphi_{in}(\Delta\omega)} = 2\frac{\left\langle |V_n|^2 \right\rangle}{|V_{01}|^2}$$

Finally we obtain a white phase-noise spectral density in a 1-Hz bandwidth

$$S_{\Delta\varphi_{in}} = 2\frac{\left\langle |I_n|^2 \right\rangle}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2 = 2\frac{S_{I_n}}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2$$

Determination of the oscillation frequency

In order to find the oscillation frequency, by setting $I_n=0$ and opening the feedback loop, the figure below is obtained



We have:

$$G_{M0}V_{ext}=I_{out} \text{ and } -\frac{I_{out}}{C}=V_1$$

The open-loop gain can be written as: $\tilde{G}=\frac{V_1}{V_{ext}}=-\frac{G_{M0}}{C}$

Where $\tilde{G}$ denotes the complex gain in the frequency domain

Determination of the oscillation frequency (continued)

Oscillation conditions are fulfilled for $V_1 = V_{ext}$. It follows that:

$$\tilde{G}(\omega_0) = 1 = -\frac{G_{M0}}{C} = -\frac{G_{M0}}{C_0}$$

Note that from the above expression C_o must be real and negative

Let $C = C_R + jC_I$,

The oscillation frequency is determined by :

$$C_I(\omega_0) = 0$$

and then $C_R(\omega_0) = C_0 = -G_{M0}$

Determination of the "loaded Q factor of the oscillator"

The loop gain is written as :

$$\tilde{G} = |G| e^{j \cdot \varphi} = -\frac{G_{M0}}{C}$$

At the oscillation frequency ω_o

$$|G|_{\cdot(\omega_0)} = 1$$

$$\varphi(\omega_0) = 0$$

From $\tilde{G} = |G| e^{j \cdot \varphi} = -\frac{G_{M0}}{C}$, the slope $\frac{d\varphi}{d\omega}$ of the loop gain can be obtained at the oscillation frequency as:

Determination of the "loaded Q factor of the oscillator" (continued)

$$\left.\frac{d\varphi}{d\omega}\right|_{\omega_0} = -\frac{C'_I}{C_0} \qquad \text{with} \qquad C'_I = \left.\frac{dC_I}{d\omega}\right|_{\omega_0}$$

We denote the "oscillator loaded Q factor" $Q_{Loscill}$ as

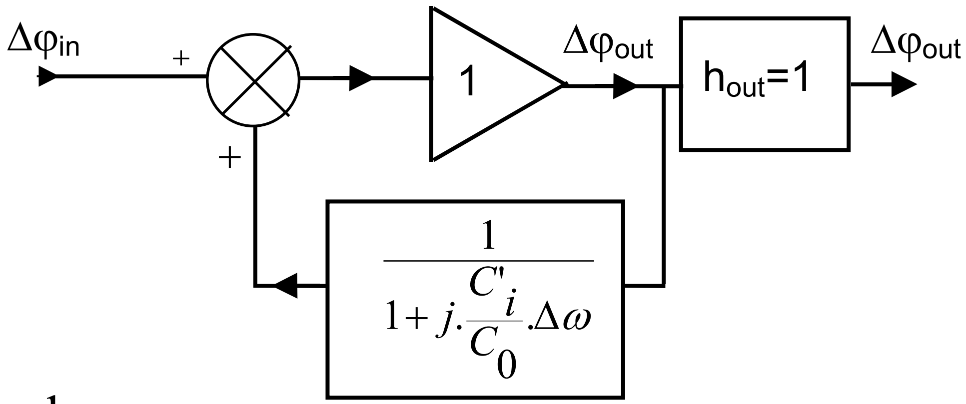
$$\boxed{Q_{Loscill}(\omega_o) = \frac{\omega_o}{2}\left|\left.\frac{d\varphi}{d\omega}\right|_{\omega_o}\right| = \frac{\omega_o}{2}\left|\frac{-C'_I}{C_0}\right|}$$

- Note as a general rule that the "oscillator loaded Q factor" $Q_{Loscill}$ does not coincide with the loaded Q factor of the passive circuit.
- It coincides only for some elemental feedback networks.

The relevant coefficient for the calculation of the output phase noise: $Q_{Loscill}$ is proportional to the group delay of the feedback path.
It is directly related to the phase-frequency relationship of the oscillator loop gain

Determination of the output phase noise

The following normalized representation of the feedback oscillator can be deduced



We have successively:

$$\Delta\varphi_{in} = \Delta\varphi_{out}\left(1-\frac{1}{1+j\frac{C'_I}{C_o}.\Delta\omega}\right), \qquad \Delta\varphi_{out} = \Delta\varphi_{in}\left(1\text{-}\,j\frac{C_0}{C'_I.\Delta\omega}\right), \qquad S_{\Delta\varphi_{out}} = S_{\Delta\varphi_{in}}\left(1+\left(\frac{C_0}{C'_I}\right)^2\frac{1}{\Delta\omega^2}\right)$$

Explicitly, we obtain

$$S_{\Delta\varphi_{out}} = 2\frac{\left\langle |I_n|^2\right\rangle}{|V_{01}|^2}\left|\frac{A_0}{C_0}\right|^2\left(1+\left(\frac{C_0}{C'_I}\right)^2\frac{1}{\Delta\omega^2}\right)$$

Determination of the output phase noise (continued)

The near carrier frequency, $S_{\Delta\varphi out}$ becomes

$$S_{\Delta\varphi_{out}} = 2\frac{\left\langle |I_n|^2 \right\rangle}{|V_{01}|^2}\left|\frac{A_0}{C'_I}\right|^2 \frac{1}{\Delta\omega^2}$$

These equations must now be compared to the corresponding Leeson formulas recalled below:

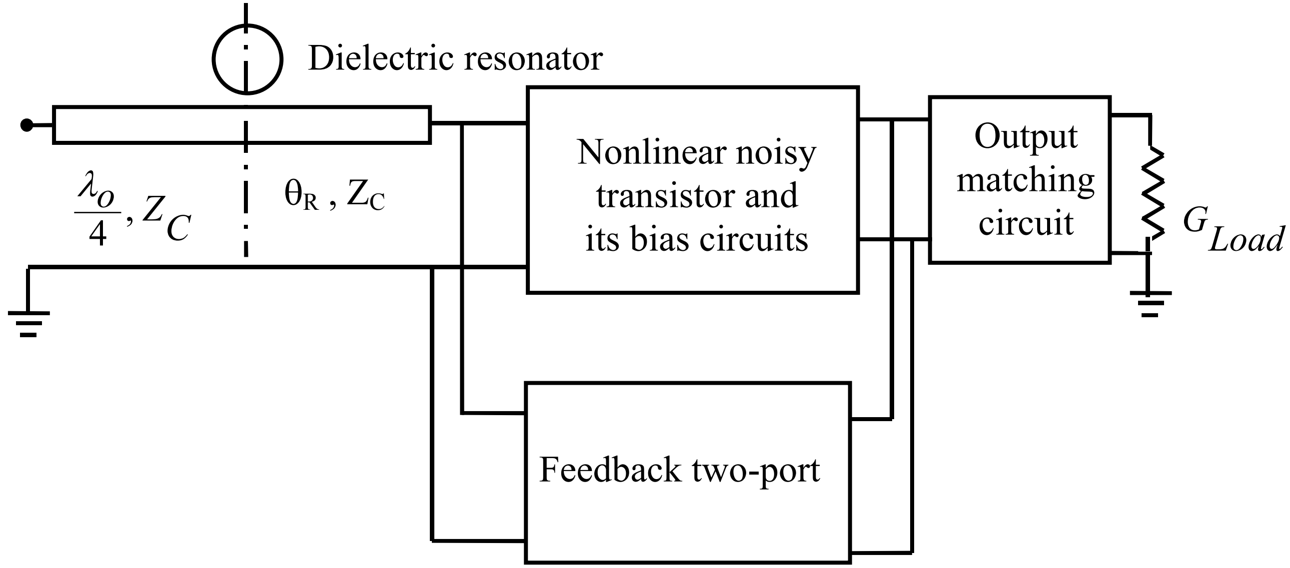
$$S_{\Delta\varphi_{out}} = S_{\Delta\varphi_{in}}\left(1+\left(\frac{\omega_0{}^2}{4Q^2_{Leeson}\Delta\omega^2}\right)\right)$$ and near carrier frequency $$S_{\Delta\varphi_{out}} = S_{\Delta\varphi_{in}}\left(\frac{\omega_0{}^2}{4Q^2_{Leeson}\Delta\omega^2}\right)$$

By comparison with the previous equations we have

$$Q_{Leeson(\omega_o)} = \frac{\omega_o}{2}\left|\frac{d\varphi}{d\omega}\right|\Bigg|_{\omega_o} = \frac{\omega_o}{2}\left|\frac{-C'_I}{C_0}\right| = Q_{Loscill(\omega_o)}$$

The hidden parameters included in the Leeson formula, may result in a $Q_{Loscill}$ different from the loaded Q factor Q_L of the feedback tank by more than one order of magnitude.

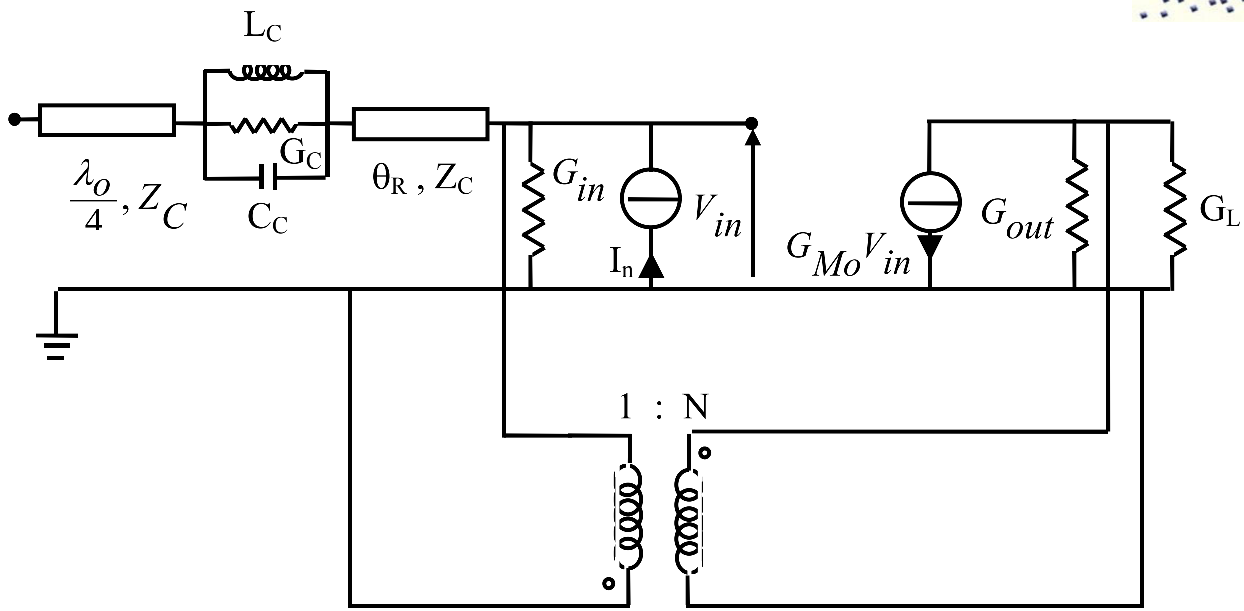
In order to account for such a possible difference, let us look at the following example



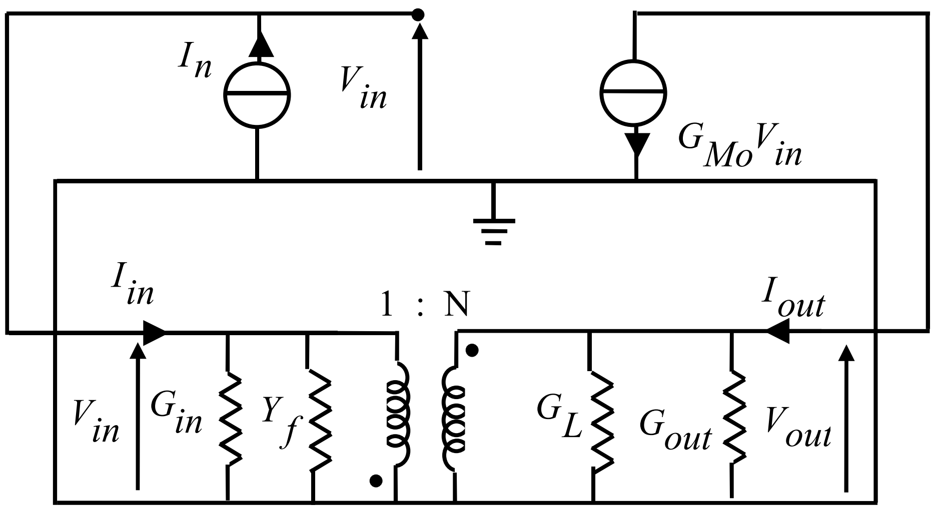
The circuit includes a dielectric resonator coupled by means of a lossless transmission line of characteristic impedance , and electrical length θ_R at the oscillation frequency.

The purpose of introducing the line is to allow variation in the loaded Q factor of the oscillator (without simultaneously varying the resonant frequency or, at the first order, the loaded Q of the passive circuit) by the varying length θ_R.

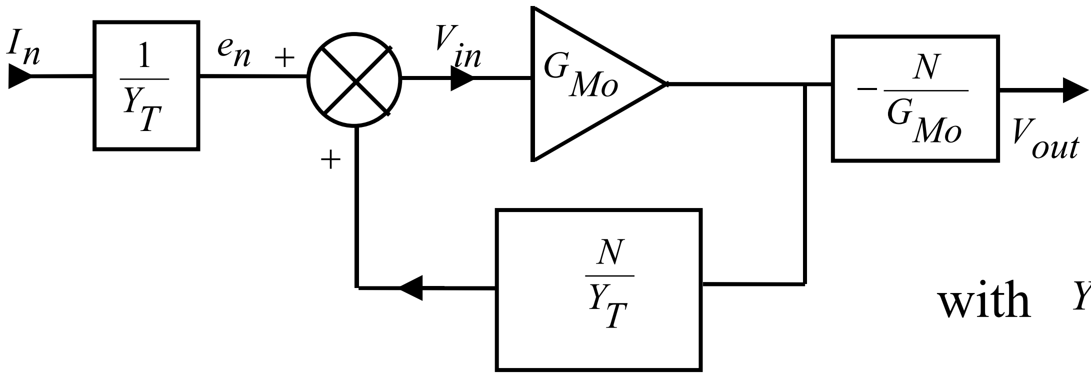
A simplified linear schematic



with the electrical equivalent circuit



A closed loop representation can be deduced



with $Y_T = Y_f + G_{in} + N^2\left(G_{out} + G_L\right)$

Y_f is the admittance of the whole resonator circuit brought back by the coupling line to the transistor input port.

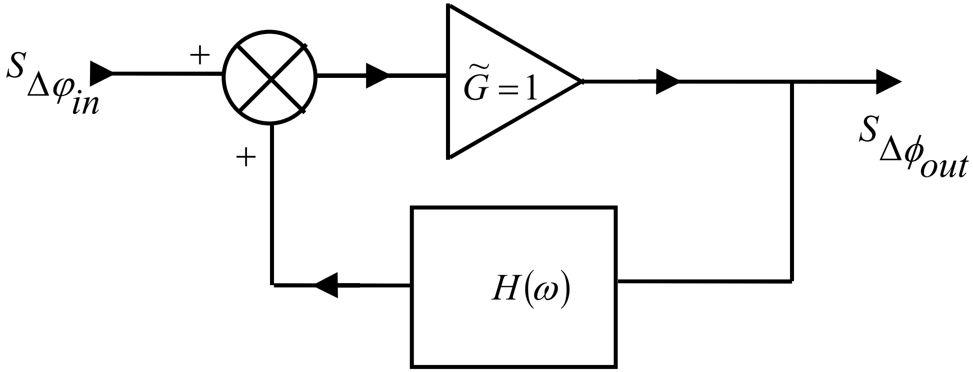
Around the oscillation frequency, a straightforward calculation gives

$$Y_f = G_c\left\{1 + j\frac{\left[2C_c - \frac{\pi\, G_c}{2\omega_o}\right]}{G_c}\cos(2\theta_o)\Delta\omega\right\}$$

The term: $2C_c$ is due to the lumped resonator, and $\frac{\pi\, G_c}{2\omega_o}$ is due to the open circuit stub

Phase noise calculation

One can now represent the oscillator around the oscillation frequency ω_o by the normalized closed loop representation of the figure below



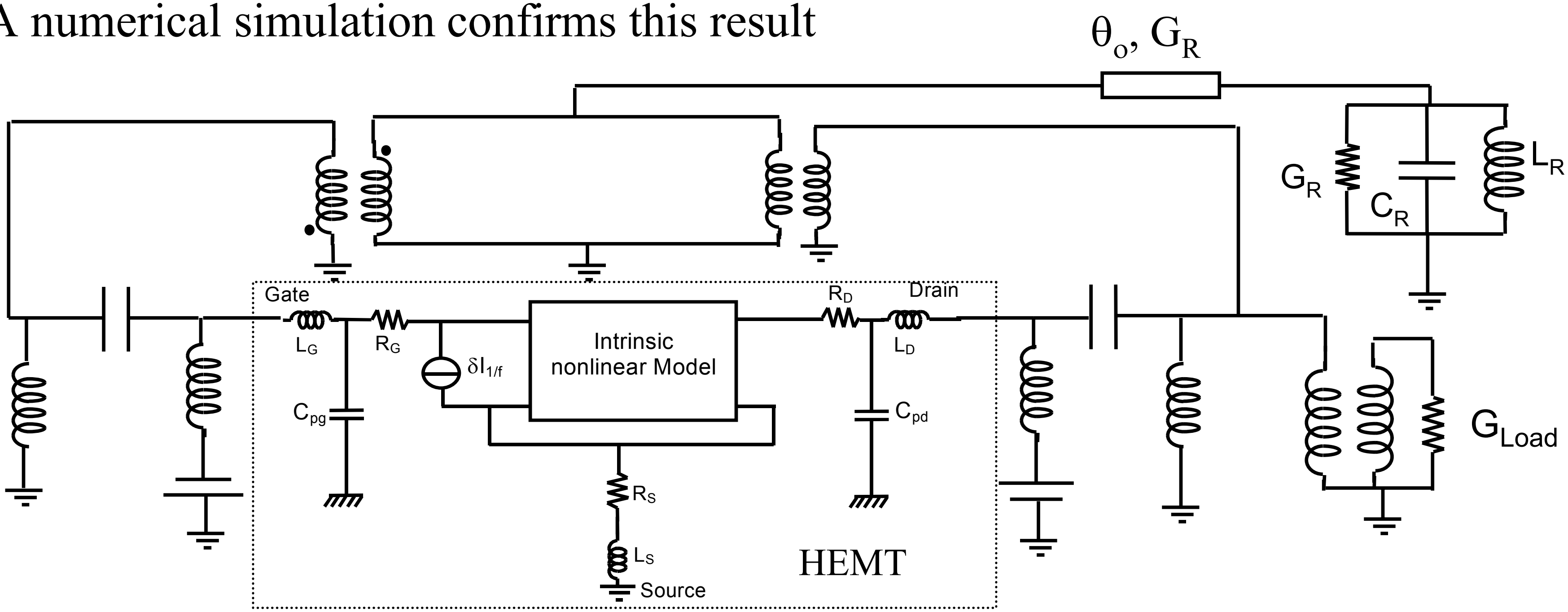
The normalized transfer function around the oscillation frequency is written as:

$$H(\Delta\omega)=\frac{1}{1-j\frac{d\varphi}{d\omega}\Delta\omega}=\frac{1}{1+j\left[\frac{2.C_C-\frac{\pi G_C}{2\omega_o}}{G_{tot}}\right]\cos(2\theta_o)\Delta\omega}$$

The output phase noise is calculated as $$S_{\Delta\varphi_{out}} = S_{\Delta\varphi_{in}}\left(1+\left[\frac{G_{tot}}{2C_c - \frac{\pi G_c}{2\omega_o}}\right]^2 \frac{1}{\cos^2(2\theta_o)\Delta\omega^2}\right)$$

From this equation, it can be shown that $S_{\delta\varphi out}$ is periodic with θ_o

A numerical simulation confirms this result

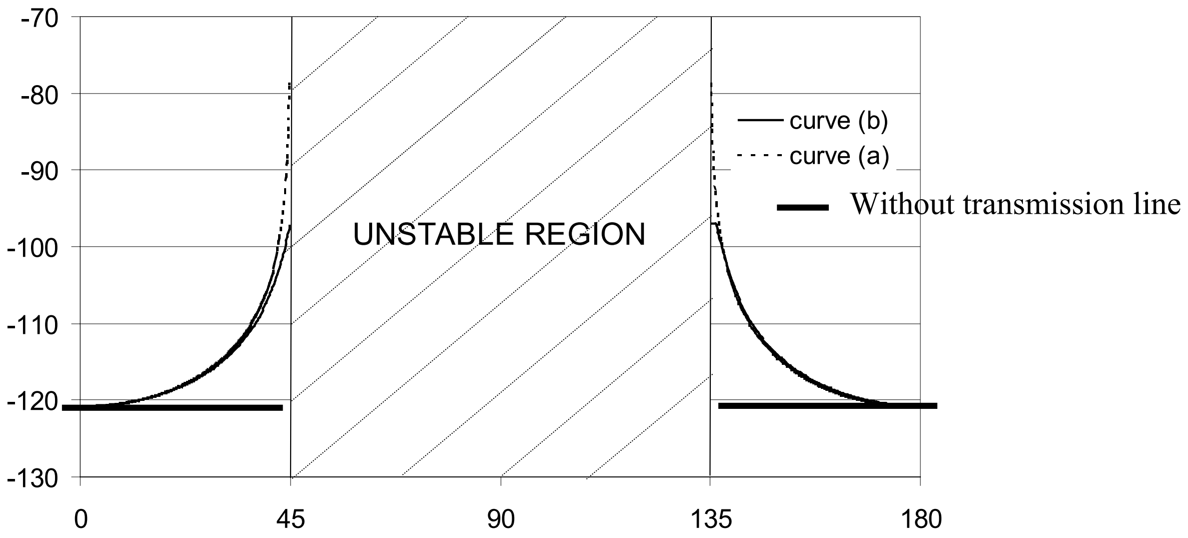


Phase noise has been successively simulated with two noise sources : a white noise source and 1/f noise source.

As expected, a phase noise roll-off, respectively of –20dB/dec and –30 dB/dec versus frequency offset from carrier is obtained.

Besides, for a fixed frequency offset, both noise sources give similar curves as a function of the electrical length: θ_o

This fig. shows the results obtained at 100 kHz from carrier, with the white noise source: curve (a) is the analytical result and curve (b) is the simulation result.



Application of the Leeson formula gives a phase noise of -121 dBc practically constant as a function of θ_o. Note that the unloaded Q factor of the resonator used was :Q_o=3000

Note:

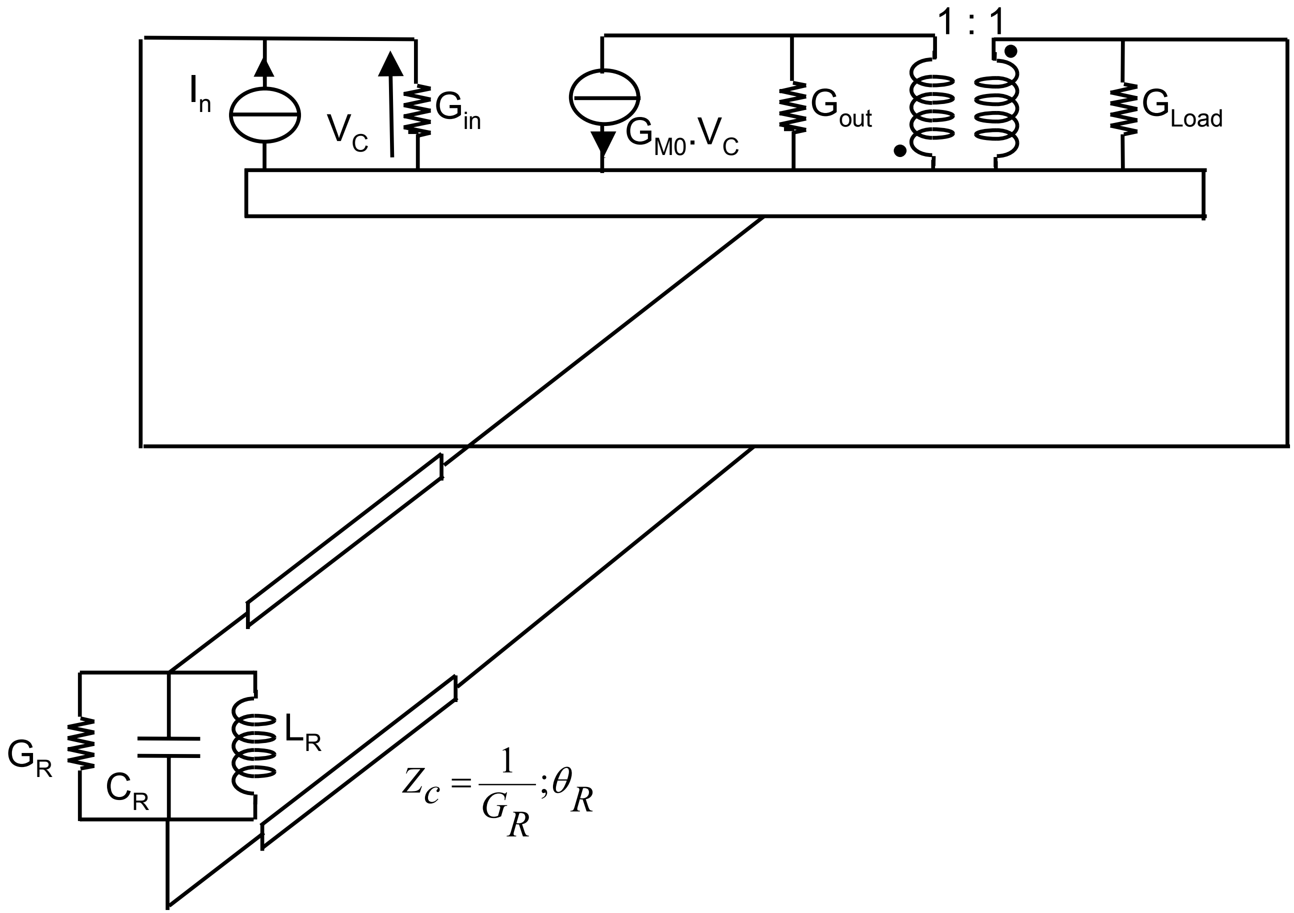
- The Q factor of the oscillator circuit defined as :

$$Q_{Loscill}(\omega_o) = \frac{\omega_o}{2} \left\| \frac{d\varphi}{d\omega} \right\|_{\omega_o}$$

Is not, as a general rule, equal to the Q factor of the circuit defined as:

$$Q_{circuit}(\omega_o) = \omega_o \frac{\overline{\varepsilon}}{\overline{P}}$$

As an example, let us consider the following simple oscillator circuit:



The energy stored is :

- In the resonator coupling line

$$\bar{\varepsilon}_{line} = \frac{1}{2}\frac{\theta_o}{Z_c\omega_o}\left|V_c(\omega_o)\right|^2$$

- In the resonator :

$$\bar{\varepsilon}_{\text{resonator}} = \frac{1}{2}C_R\left|V_c(\omega_o)\right|^2$$

- Power dissipated :

$$\bar{P} = \frac{1}{2}\left[G_{in} + G_L + G_{out}\right]\left|V_c(\omega_o)\right|^2 = \frac{1}{2}G_{tot}\left|V_c(\omega_o)\right|^2$$

Then,

$$Q_L(\omega_o) = \omega_o\frac{\bar{\varepsilon}_{tot}}{\bar{P}} = \omega_o\left[\frac{C_R + \dfrac{\theta_o}{Z_c\omega_o}}{G_{tot}}\right]$$

The loop gain phase slope around ω_o is

$$\left|\frac{d\varphi}{d\omega}\right|_{\omega o} = \frac{2C_R}{G_{tot}}\left|\cos(2\theta_o)\right|$$

and

$$Q_{oscill} = \frac{\omega o}{2}\left|\frac{d\varphi}{d\omega}\right|_{\omega o} = \omega_o \frac{C_R}{G_{tot}}\left|\cos(2\theta_o)\right|$$

Q_{oscill} is different from Q_L,

However for $\theta_o = 0$ $\Longrightarrow$ $Q_{oscill} = Q_L$

Modeling of Semi-conductor devices

- Large signal models
- Noise sources

–Large signal electrical models must be physically based in order to accurately take into account the interactions between the noise sources and the nonlinear elements in the semi-conductor devices.

–The low frequency noise sources up converted near the oscillation frequency must be accurately localized in the semi-conductor devices.

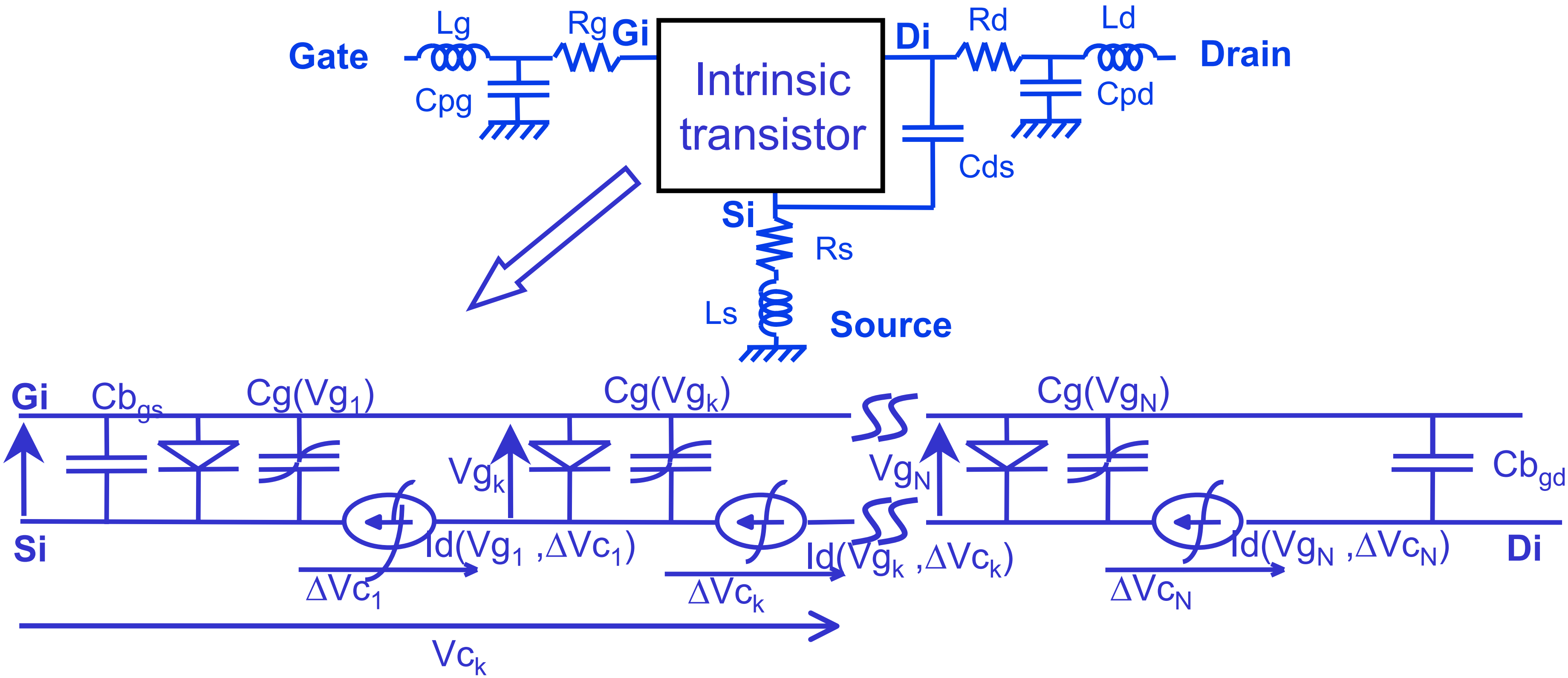
The counter example

- Let us suppose I_{n1} and I_{n2} are two low frequency noise sources representing the L.F noise sources extracted from the transistor

If this model was physically accurate, it would be sufficient to bias the transistor through two choke inductances in order to eliminate all possible up-conversions

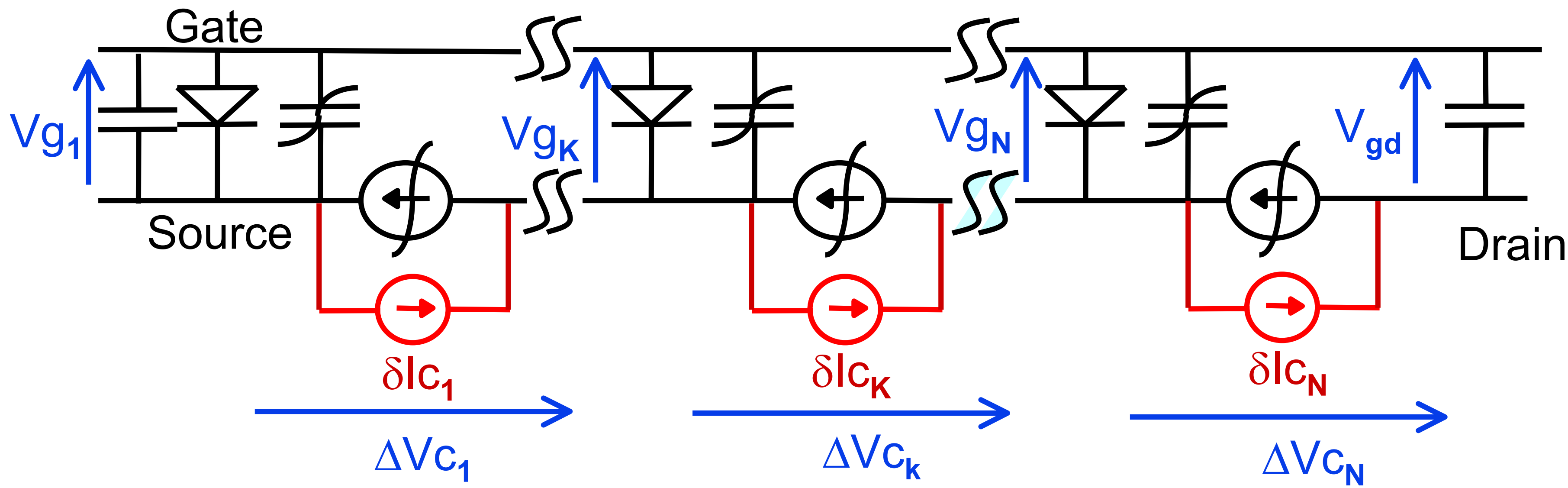
Unfortunately in nonlinear design, the noise sources must be localized exactly where they are generated.

An example of nonlinear distributed model of HEMT



In field-effect transistors, accurate modeling of the interaction between steady-state large signal and low-frequency noise sources distributed along the source-drain channel, requires a distributed model.

For this purpose, a non-linear, non-uniform, distributed model of FETs has been developed



Nonlinear HEMT Model including the L.F noise sources of the channel

- The channel is considered as a non-uniform, non-linear, active transmission line with N unit cells. The number of cells depends on the channel length and the operating frequency. A good compromise for microwave devices is $10 \leq N \leq 20$.

- Every unit cell includes :
  - A non-linear gate-channel capacitance: $C=f_C(Vg_k)$ in parallel with a Schottky diode: $I = f_G(Vg_k)$. These elements are a function of their own port voltage Vg_k.
  - A non-linear channel current-controlled source: $Ic = f_I(Vg_k,\Delta Vc_k)$ which depends on the two cell-voltages: ΔVc_k and Vg_k.

- The model has been extracted for many HEMTs from different processes.

Figure 1 shows a comparison between the model and measured characteristics for a 4*0.25*50 µm PHEMT modeled with 10 unit-cells.

- It must be pointed out that :

- The model is fully extracted from measurements.
- The functions fc, f_G, f_I are identical for all unit cells.
- The model reproduces the non-uniform non-quasi-static behavior of the transistor in a natural way.

MEASURED AND MODELED I-V CHARACTERISTIC

PH25 - 4 x 50 µm Vgso= -0.28 V, Vdso=2.51 V

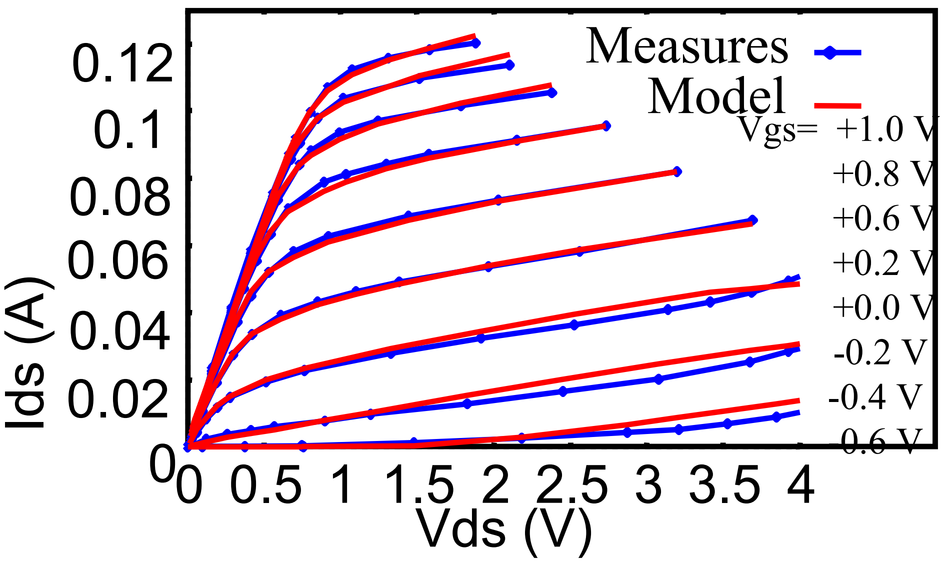
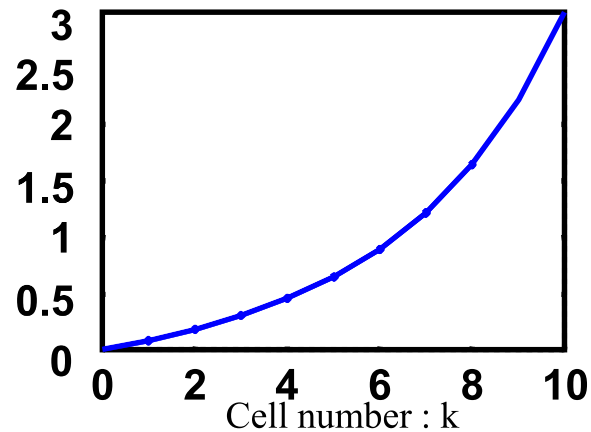
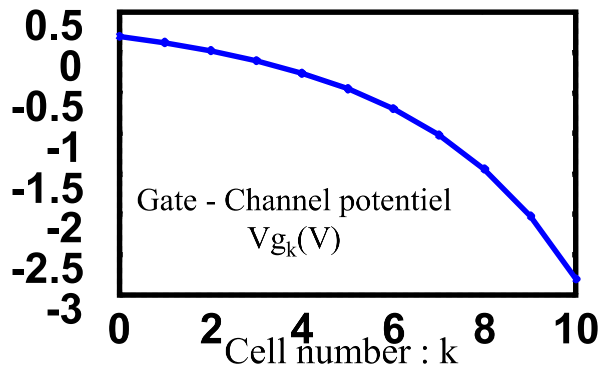


Fig. 1 : PH 25 Transistor distributed model

DISTRIBUTED MODEL : Vc_k AND Vg_k

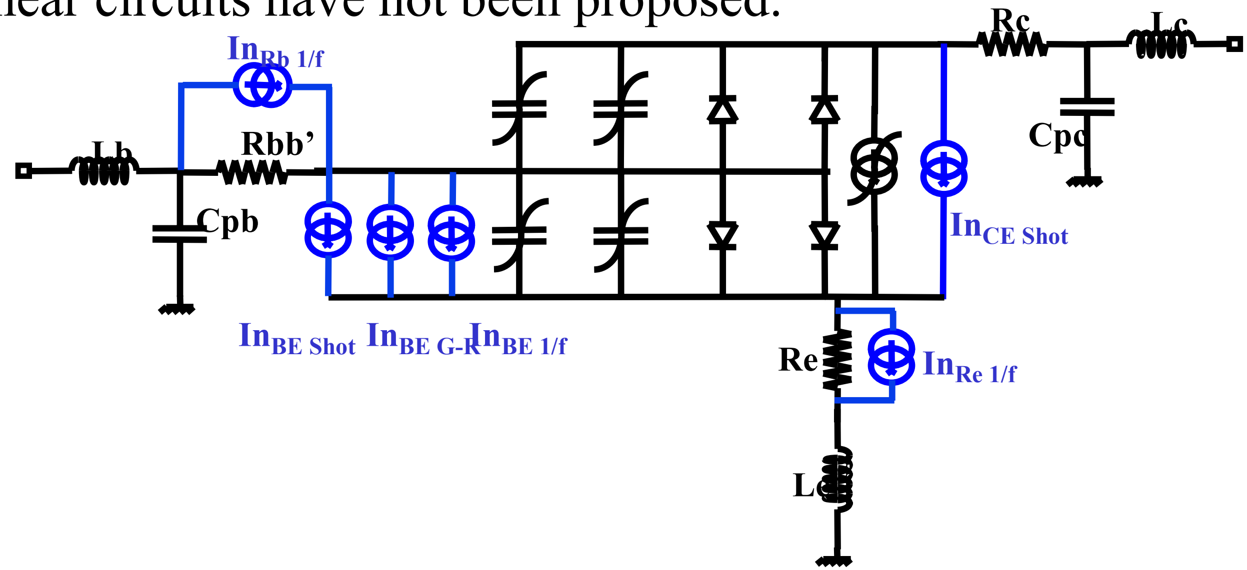




Note that such a distributed model including distributed noise sources has been recently proposed for MOSFETs

Some bipolar transistor models including a distributed circuit (for the base region), have been proposed for linear applications (Te-Winkel, Pritchard, Van der Ziel,…)

However to our knowledge, nonlinear distributed models of HBTs suitable for CAD of nonlinear circuits have not been proposed.



This figure shows an example of HBT nonlinear model including the main noise sources used for CAD of oscillator circuits.

Note that the circuit shows "possible localization of noise sources". Fortunately many of them can be neglected, according to the technology.

Noise source modeling

– Physical noise sources:

Low-frequency noise sources → G-R Noise; Fundamental 1/f noise

RF noise sources: diffusion noise → Linear resistances; FET-Channel; Shot noise in SC junctions

→Two physical sources of low-frequency noise must be considered :

- $\delta N(t)$: Random fluctuations of the carrier number : N
- $\delta\mu(t)$: Random fluctuations of the low-field mobility : μ_o

• The first one is due to traps present in the semi-conductor.

• The second is named the " fundamental 1/f noise ", and applies also to the low-frequency fluctuations of the diffusion coefficient $\delta D(t)$.

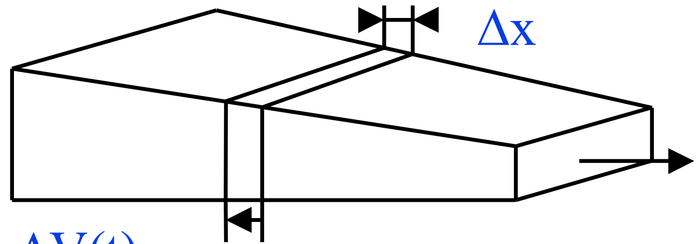
→One physical source of RF noise :

The diffusion noise gives rise to white noise sources:

- Low electric field : thermal noise
- High electric field: diffusion noise
- Noise of minority carriers in semi-conductor junctions: shot noise

Circuit-CAD oriented modeling of low-frequency noise sources in semi-conductor devices

LOCAL CURRENT- NOISE SOURCES IN HOMOGENEOUS LINEAR SAMPLES



Slice of homogeneous linear sample

- Let us consider a slice : Δx with a cross-section : A, and a carrier concentration : n_o
- the velocity of carriers due to the deterministic local applied voltage : $\Delta V(t)$ writes :

$$V_c(t) = \mu_o \frac{\Delta V(t)}{\Delta x}$$

- A deterministic current I(t) follows :

$$I(t) = q.\frac{N_o}{\Delta x}.V_c(t) = \frac{q}{\Delta x^2}.\Delta V(t).\mu_o.N_o$$ with $N_o = n_o.A.\,\Delta x$

- If low-frequency random fluctuations : $\delta N(t)$ and $\delta\mu(t)$ are present in the slice : $A.\Delta x$

→ Local Norton noise sources are given by :

$$\delta i(t) = \frac{q}{\Delta x^2}.\Delta V(t).\left[\mu_o.\delta N(t) + N_o.\delta\mu(t)\right]$$

DC+RF Deterministic applied voltage

Low-frequency random fluctuations

Low-frequency random fluctuations

Conclusion: The initial low-frequency random fluctuations : $\delta N(t)$ and $\delta\mu(t)$ are "probed" by the DC+RF deterministic applied voltage, which results in Norton noise sources with the initial low-frequency spectrum transposed around the Fourier components of the deterministic applied voltage giving rise to cyclostationary noise sources.

–RF noise:

Since Dragone, we have known that the diffusion noise with a δ correlation function is modulated by a large-signal periodic applied voltage.

- A local noise source described for a DC applied by its spectral density:

$$\left\langle \delta I^2(x) \right\rangle = 4q^2 \frac{Dn(x)}{A} \delta\left(x - x'\right) \Delta f$$

- becomes cyclostationary when a periodic large signal is applied:

$$\left\langle \delta I_k(x) \delta I_j^*(x) \right\rangle = 4q^2 \frac{DN(k-j)}{A} \delta\left(x - x'\right) \Delta f$$

Where $N(k-j)$ is the $(k-j)$ th harmonic of n(x,t)

In other words:

In the presence of a large signal periodic electric field:

- the low frequency current noise sources
- and the RF current noise sources

Are modulated and create upper and lower side-bands around every harmonic component of the large signal periodic electric field.

An external DC Applied Voltage gives rise to:

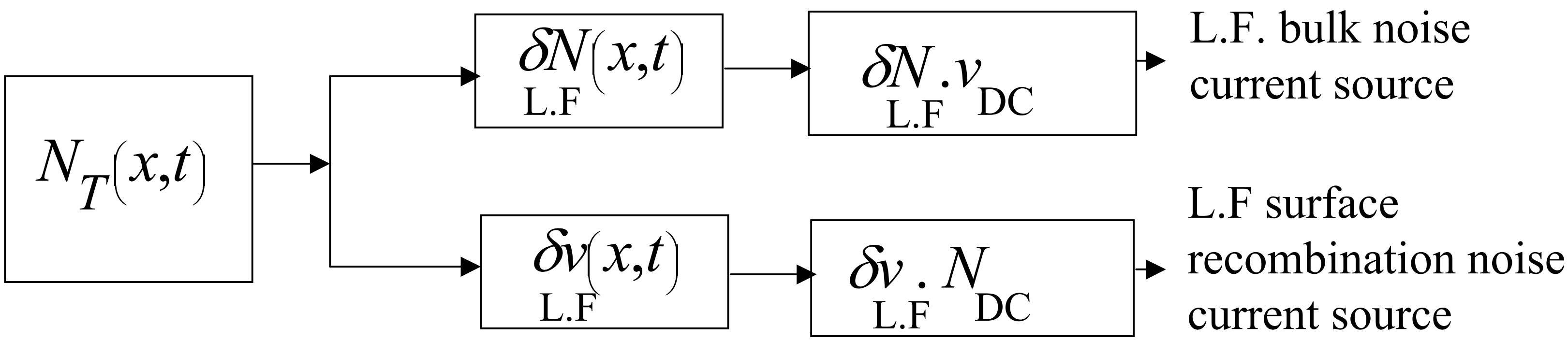
- DC local current density into the S.C. device:

$$J_o(x) = q.N(x).v(x) \longrightarrow \text{DC}$$

L.F. random fluctuations of traps occupancy : $N_T(x,t)$ gives:

- L.F. random fluctuations of free carriers : $\delta N(x,t) \longrightarrow$ bulk
- L.F. random fluctuations of velocity recombination : $\delta v(x,t) \rightarrow$ surface

It results two main generations:



An external Large Signal Applied Voltage gives rise to:

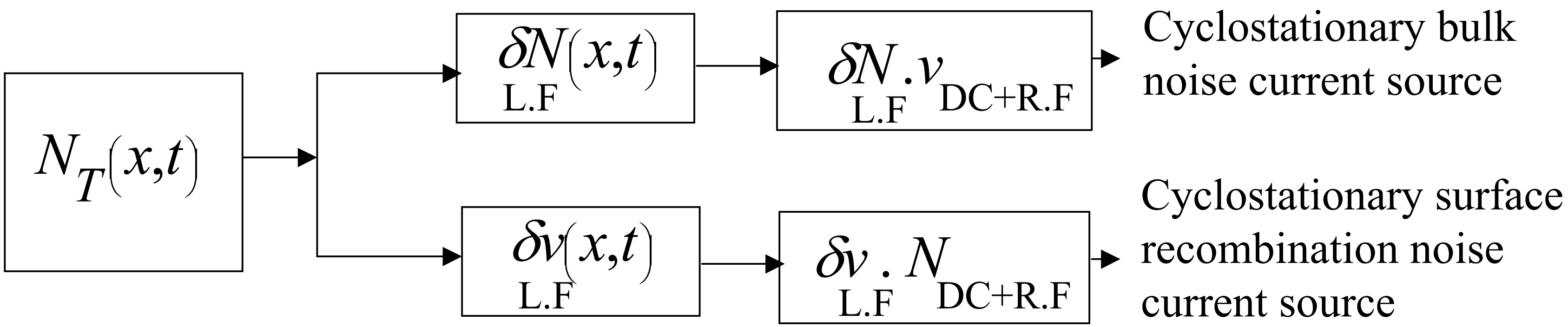
- Large signal local current density into the S.C. device:

$$J(x,t) = q.N(x,t).v(x,t) \longrightarrow \text{DC and RF components}$$

L.F. random fluctuations of traps occupancy : $N_T(x,t)$ gives:

- L.F. random fluctuations of free carriers : $\delta N(x,t) \longrightarrow$ bulk
- L.F. random fluctuations of velocity recombination : $\delta v(x,t) \rightarrow$ surface

It results two main generations:



Simulation tools

A coherent set of robust and reliable simulation-tools is required in order to :

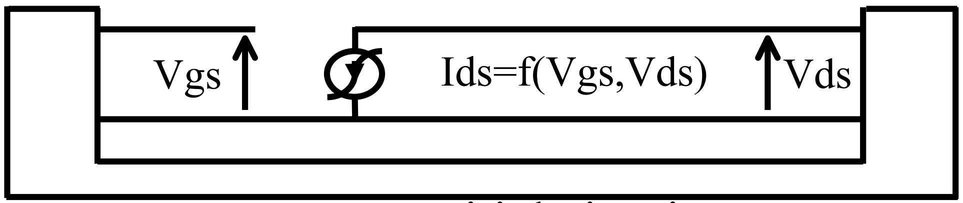
- Find the steady-state oscillation-frequency and variables at every node of the circuit.
- Perform a linear/non-linear stability analysis and find possible spurious frequencies.
- Calculate AM and PM noise spectrum.

→ Steady-state analysis is generally performed using the well-established harmonic-balance method.

A simplified NL local stability analysis:

→ The linear/non-linear stability analysis may now be easily performed with commercially available software packages.

The figure below shows the principle of the non-linear open loop concept applied to a HEMT oscillator circuit, for stability analysis purposes.

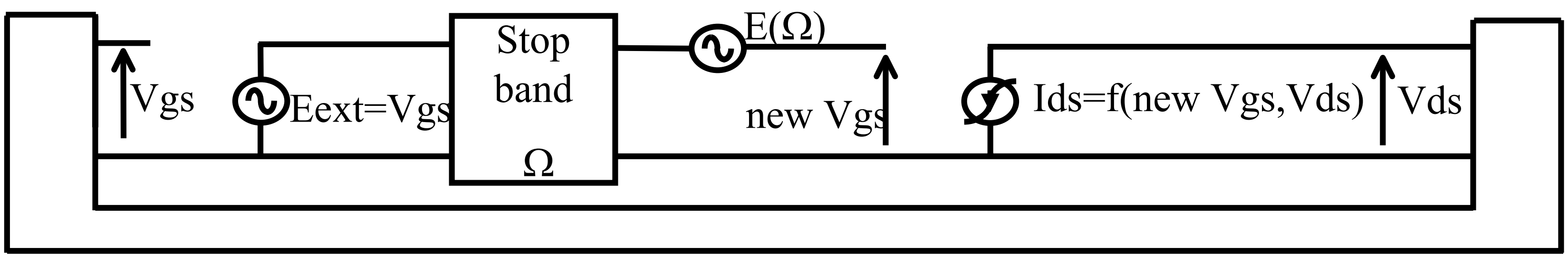


Initial circuit

Stop band filter

$\omega = \Omega$

$E(\Omega)$: small signal perturbation



Modified transistor model for stability analysis

The open loop gain $\tilde{G}(\Omega) = \dfrac{V_{gs}(\Omega)}{E(\Omega)}$ is calculated and its module and phase are plotted as a function of frequency.

Note that these plots are equivalent to the plot drawn in the linear case for the determination of the oscillation frequency

(See section: Modern linear design approach)

Steady-state analysis

- Modern harmonic balance software packages include:
    - Modified nodal analysis formulation to describe the circuit
    - Krylov subspace methods to solve the network matrices

-Conditions for a successful steady-state simulation:
- High number of spectral components

  ⟶ High number should lead to high accuracy, but

- Are the models of passive elements and active devices valid at very high frequencies ?

Solution : Short-circuit at the active device ports for $f > Nf_o$
N: depending on the foundry models.

- Are the number of samples of the time-domain state variables sufficient?

Solution : Accurate results need oversampling

Conclusion:

Special care is needed in choosing the number of spectral components as well as the number of time domain samples.

(See example below)

Simulation of phase noise in oscillators

- Accurate simulation needs to take into account:

- All kind of noise sources localized at every node of the circuit:

  - at low frequency
  - or RF frequency

- linear, and cyclostationary noise sources.

- must take also into account the low-frequency dynamics of the oscillator circuit and bias network (see example below)

- Accurate modeling of the noise sources:

- Current models are generally not sufficient

There is a confusion between :

- Models valid only for linear applications

- Models whose validity extends to nonlinear applications

-The localization of the noise sources in the nonlinear device models is still only approximated.

Oscillator phase noise calculation in the frequency domain by means of conversion matrices formalism

- output waveform:

$$V(t) = V_o\left(1 + \frac{\Delta v(t)}{V_o}\right)\cos\left(\omega_o t + \varphi_o + \Delta\varphi(t)\right)$$

V_o	Peak amplitude of the carrier ω_o
φ_o	A constant phase
$\Delta v(t)$	Amplitude noise modulation
$\Delta\varphi(t)$	Phase noise modulation

- At a frequency offset Ω, from carrier:

AM and PM noise modulations are expressed as pseudosinusoids

$$\Delta v(t) = \Delta V_O \cos(\Omega t + \Phi_A)$$

$$\Delta \varphi(t) = \Delta \varphi_O \cos(\Omega t + \Phi_\phi)$$

Where $\left\langle \Delta v(t)^2 \right\rangle$ and $\left\langle \Delta \varphi(t)^2 \right\rangle$ denote their associated spectral densities; $\Delta V_O, \Delta\varphi_O, \phi_A, \phi_\phi$ are random variables.

We can write V(t) as :

$$V(t) = V_O \left[\cos(\omega_O t + \varphi_O) + \frac{V_\Delta}{V_O} \cos((\omega_O - \Omega)t + \varphi_\Delta) + \frac{V_\Sigma}{V_O} \cos((\omega_O + \Omega)t + \varphi_\Sigma) \right]$$

Let's write $\dfrac{\Delta V_o e^{j\phi_A}}{V_o} = \dfrac{\Delta\tilde{V}}{V_o}$

$$\Delta\varphi_o e^{j\phi_\varphi} = \Delta\tilde{\varphi}$$

$$V_\Delta e^{j\phi_\Delta} = \Delta\tilde{V}_\Delta \qquad \text{where} \qquad V_\Delta = V(\omega_o - \Omega)$$

$$V_\Sigma e^{j\phi_\Sigma} = \Delta\tilde{V}_\Sigma \qquad \qquad V_\Sigma = V(\omega_o + \Omega)$$

We obtain :

$$\text{AM noise} = \frac{\mathrm{P}_{\mathrm{AMtot}}}{\mathrm{P}_{\mathrm{carrier}}} = \frac{\left\langle \left|\Delta\tilde{V}\right|^2 \right\rangle}{V_o^2} = S_A = \frac{\left\langle \left| \tilde{V}_\Delta^* e^{j\varphi_o} + \tilde{V}_\Sigma^* e^{-j\varphi_o} \right|^2 \right\rangle}{V_o^2}$$

$$\text{PM noise} = \frac{\mathrm{P}_{\mathrm{PMtot}}}{\mathrm{P}_{\mathrm{carrier}}} = \frac{\left\langle \left|\Delta\tilde{\varphi}\right|^2 \right\rangle}{V_o^2} = S_\phi = \frac{\left\langle \left| \tilde{V}_\Delta^* e^{j\varphi_o} - \tilde{V}_\Sigma^* e^{-j\varphi_o} \right|^2 \right\rangle}{V_o^2}$$

The AM and PM noise are defined as:

AM noise $\left.\frac{N}{C}\right|_{at\ \Omega\ \mathrm{dBc}} = 10\log\frac{P_{ssb}(AM)}{P_{carrier}}$ by Hertz

PM noise $\mathcal{L}_{at\ \Omega\ \mathrm{dBc}} = 10\log\frac{P_{ssb}(PM)}{P_{carrier}}$ by Hertz $= \frac{S\phi}{2}$

$\tilde{V}_{\Delta}, \tilde{V}_{\Sigma}, V_O, \varphi_O$ are calculated using the harmonic balance and its related technique: the conversion matrices formalism

RELATION BETWEEN STEADY-STATE ACCURACY (ε) AND PHASE NOISE RESULTS

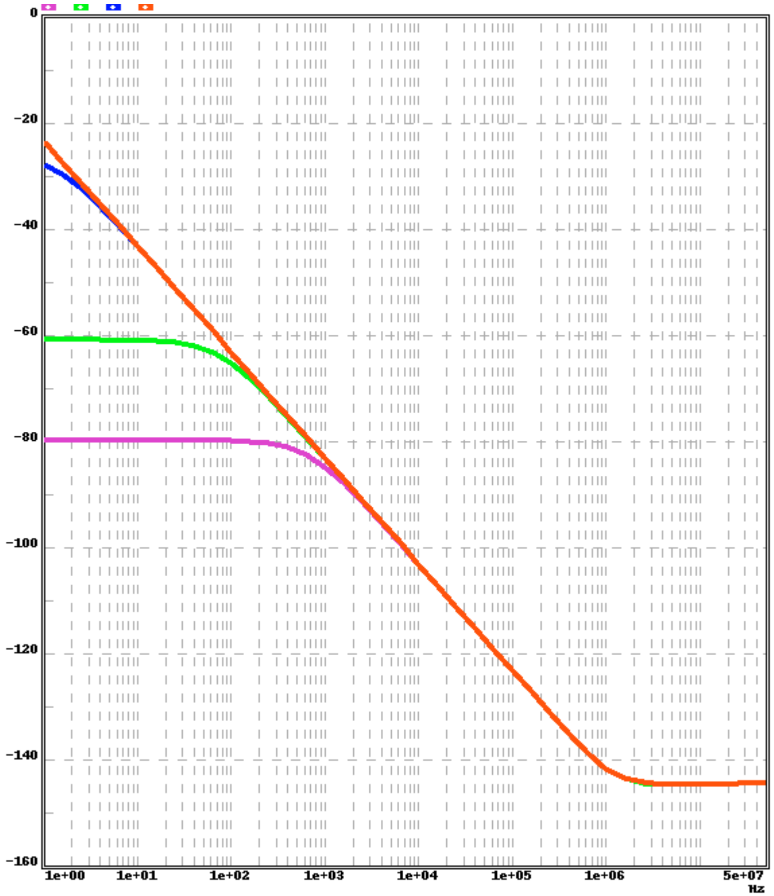
$\varepsilon = f\,(NH, Nt, \delta\omega_0)$

$\varepsilon < \varepsilon < \varepsilon < \varepsilon$

Accurate PM noise calculation is directly related to the large steady-state accuracy.

It is only a numerical problem which is solved by an appropriate choice of :

- number of harmonics
- oversampling



Conversion matrices formalism

– allows to handle large signal/small signal interactions in nonlinear devices with good accuracy

– very efficient for noise calculation in nonlinear circuits driven by a periodic/quasi-periodic large signal :

- mixers
- converters
- oscillators

- Principle of Conversion matrices

Let us consider a nonlinear element described by its constitutive equation:

$$y(t)=f\left(x(t)\right)$$

Examples: - Nonlinear conductance I/V
- Nonlinear capacitance Q/V

In small-signal applications around a bias point X_o:

$$\delta y(t)=\left.\frac{df}{dx}\right|_{X_o}\delta\left(x(t)\right)=h_o\delta\left(x(t)\right)$$

h_o : constant, time invariant

- If X_o becomes a periodic time varying bias point when a large signal of fundamental frequency ω_o is applied:

$$X_o \rightarrow X(t) = \sum_{-N}^{+N} X_k e^{jk\omega_o t}$$

Then: $$h_o \rightarrow h(t) = \sum_{-N}^{+N} H_k e^{jk\omega_o t}$$

- If the small signal $\delta x(t)$ contains an input frequency: Ω

Intermodulation components are generated at frequencies:

$$k\omega_o \pm \Omega$$

- In the frequency domain $\delta x(t)$ and $\delta y(t)$ are expanded as:

$$\delta\vec{X}(\omega)=\begin{bmatrix}\delta X^{*}(N\omega_o-\Omega)\\ .\\ \delta X^{*}(k\omega_o-\Omega)\\ .\\ \delta X(\Omega)\\ .\\ \delta X(k\omega_o+\Omega)\\ .\\ \delta X(N\omega_o+\Omega)\end{bmatrix} \quad \text{and} \quad \delta\vec{Y}(\omega)=\begin{bmatrix}\delta Y^{*}(N\omega_o-\Omega)\\ .\\ \delta Y^{*}(k\omega_o-\Omega)\\ .\\ \delta Y(\Omega)\\ .\\ \delta Y(k\omega_o+\Omega)\\ .\\ \delta Y(N\omega_o+\Omega)\end{bmatrix}$$

Then by identification with: $\delta y(t)=h(t)\delta(x(t))$

A matrix relation is found in the frequency domain:

$$\boxed{\delta\vec{Y}(\omega)=\left[H\right]\delta\vec{X}(\omega)}$$

with:

$$\left[H\right]=\begin{bmatrix} H_o & H_1^* & H_2^* & . & . & . & H_{2N}^* \\ H_1 & H_o & H_1^* & . & . & . & H_{2N-1}^* \\ H_2 & H_1 & H_o & . & . & . & H_{2N-2}^* \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ H_{2N} & H_{2N-1} & . & . & . & . & H_o \end{bmatrix}$$

$\left[H\right]$ is the conversion matrix of the nonlinear element

- The variable δX(ω) and the resulting function δY(ω) at each mixing frequency are related by the *conversion matrix*.
- Since the relation between $\delta\vec{X}(\omega)$ and $\delta\vec{Y}(\omega)$ is linear (Matrix form), it results that the nonlinear devices appear as a linear periodic time varying element:

$$\underbrace{y(t)=f(x(t))}_{\text{Nonlinear element}} \quad \underset{\text{becomes}}{\rightarrow} \quad \underbrace{\delta\vec{y}(\omega)=[H]\,\delta\vec{x}(\omega)}_{\text{L.P.T.V. element}}$$

Noise analysis can be performed using the conversion matrices formalism when the large signal is applied at the oscillation frequency ω_o and its harmonic frequencies.

For example, we recall the phase noise expression :

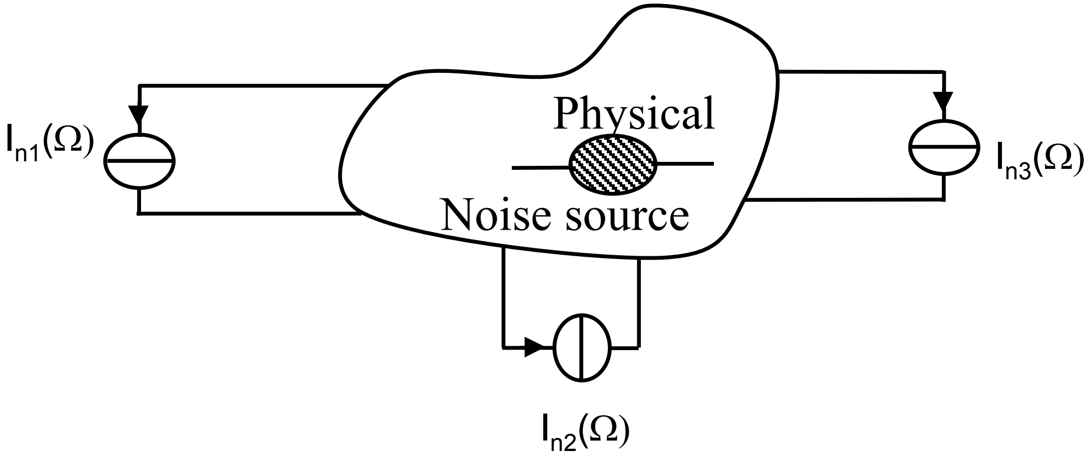
$$S_{\phi} = \frac{\left\langle \left| \tilde{V}_{\Delta}^{*} e^{j\varphi_o} - \tilde{V}_{\Sigma}^{*} e^{-j\varphi_o} \right|^2 \right\rangle}{V_o^2}$$

-first : V_o, φ_o of the noiseless oscillator are computed by harmonic balance.

-secondly : $\tilde{V}_{\Delta}$ at $\omega_o - \Omega$ and $\tilde{V}_{\Sigma}$ at $\omega_o + \Omega$ due to internal noise sources, are computed by the conversion matrices method

Noise correlation matrix concept

- Introduced for several linear noise sources by H.A Haus and R.B. Adler : 1956



- One internal physical source at a frequency Ω gives rise to several equivalent external correlated noise sources at the same frequency Ω

The correlation matrix at the given frequency Ω writes:

$$C(\Omega)=\begin{bmatrix} \left\langle \left| I_{n1}(\Omega) \right|^2 \right\rangle & \left\langle I_{n1}(\Omega) I^*_{n2}(\Omega) \right\rangle & \left\langle I_{n1}(\Omega) I^*_{n3}(\Omega) \right\rangle \\ \left\langle I^*_{n1}(\Omega) I_{n2}(\Omega) \right\rangle & \left\langle \left| I_{n2}(\Omega) \right|^2 \right\rangle & \left\langle I_{n2}(\Omega) I^*_{n3}(\Omega) \right\rangle \\ \left\langle I^*_{n3}(\Omega) I_{n1}(\Omega) \right\rangle & \left\langle I^*_{n3}(\Omega) I_{n2}(\Omega) \right\rangle & \left\langle \left| I_{n3}(\Omega) \right|^2 \right\rangle \end{bmatrix}$$

The correlation matrix of several linear noise sources describes the auto-correlation and cross-correlation between them at a frequency Ω.

Noise correlation matrix concept (continued)

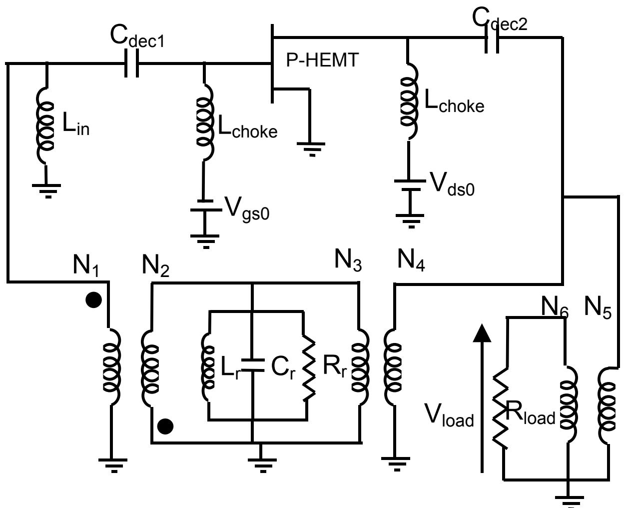
- Extension to cyclostationary noise sources:

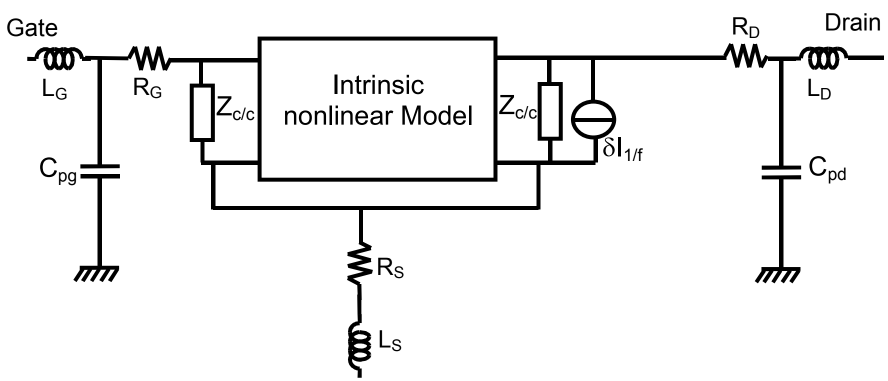
  - The correlation matrix of a cyclostationary noise source describes the auto-correlation and cross correlation between the noise components at different frequencies : $k\omega_o \pm \Omega$ and $p\omega_o \pm \Omega$, of a single noise source.

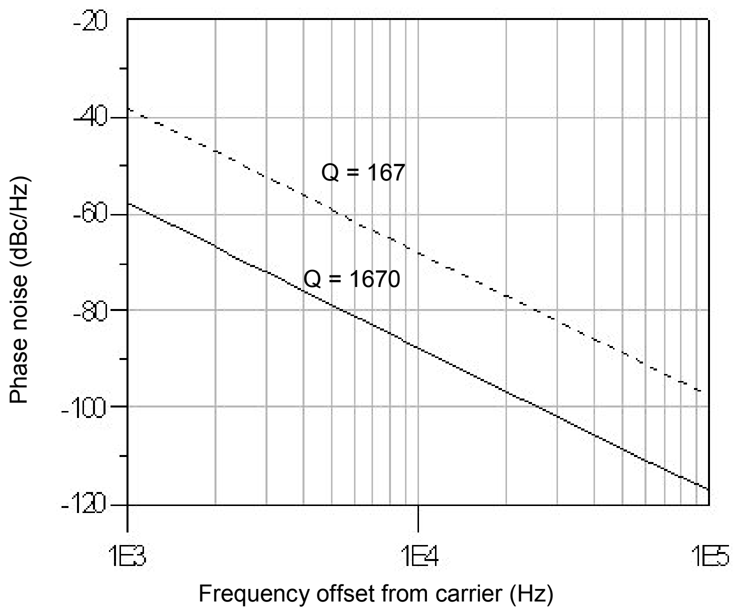
  - The correlation matrix of several noise sources describes the auto-correlation and cross correlation of noise components at different frequencies of several correlated cyclostationary noise sources.

Two examples of phase noise calculation:

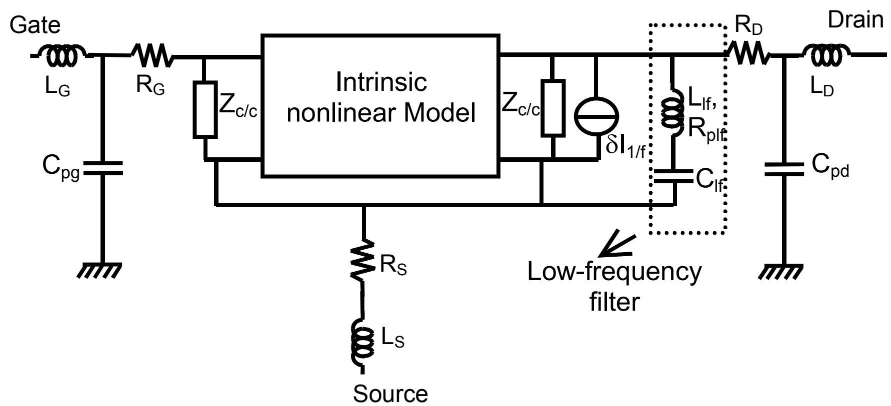
A basic oscillator circuit :

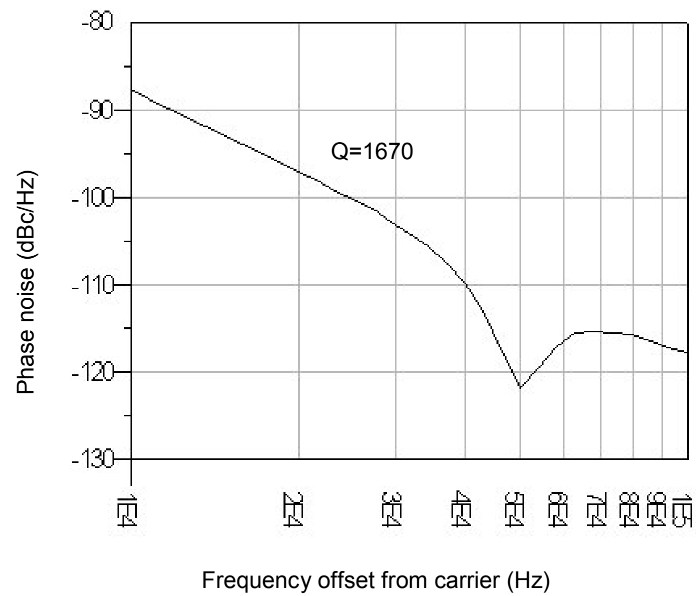






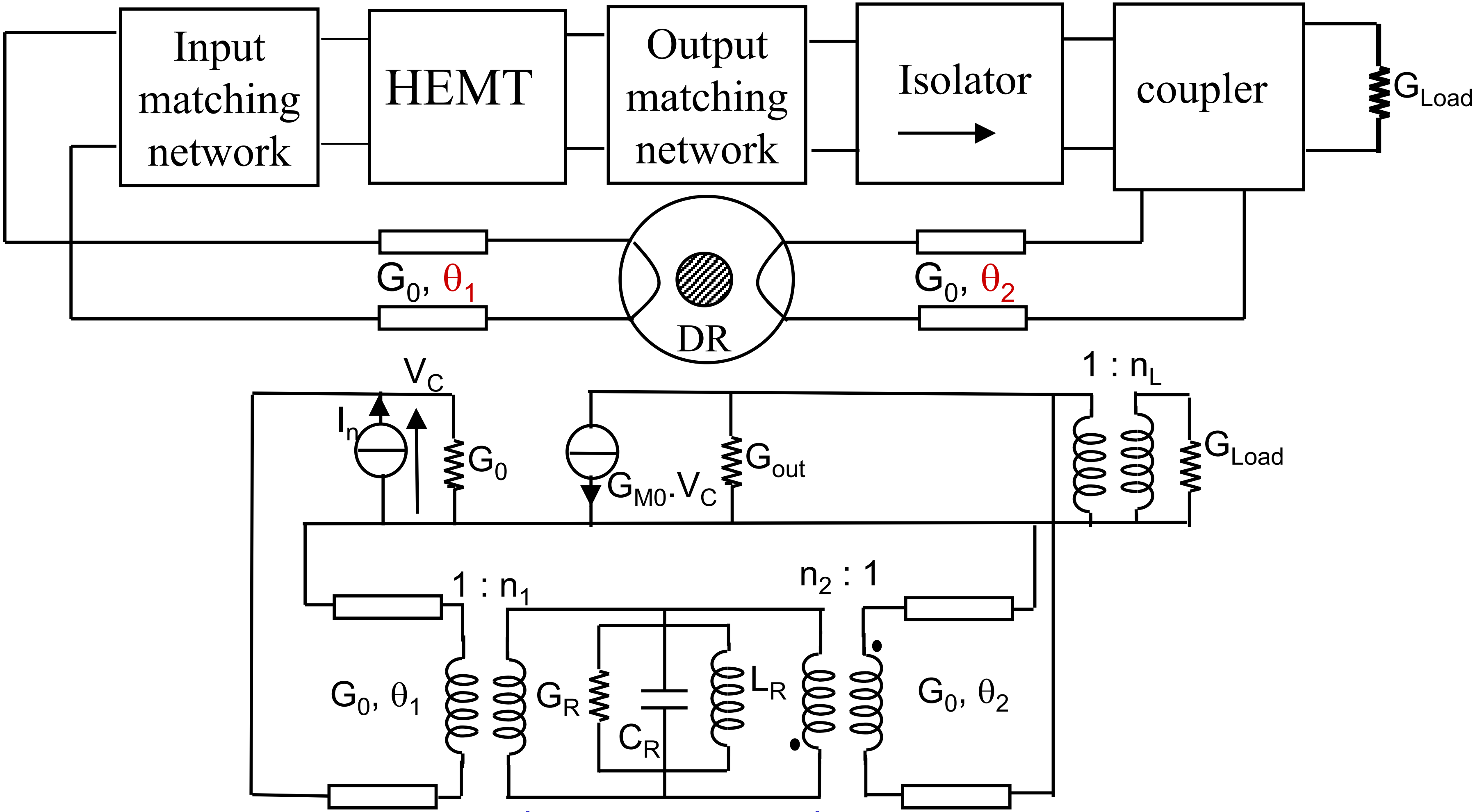
An oscillator circuit showing the influence of the low-frequency dynamics of the bias network





Practical examples

- Breadboard example

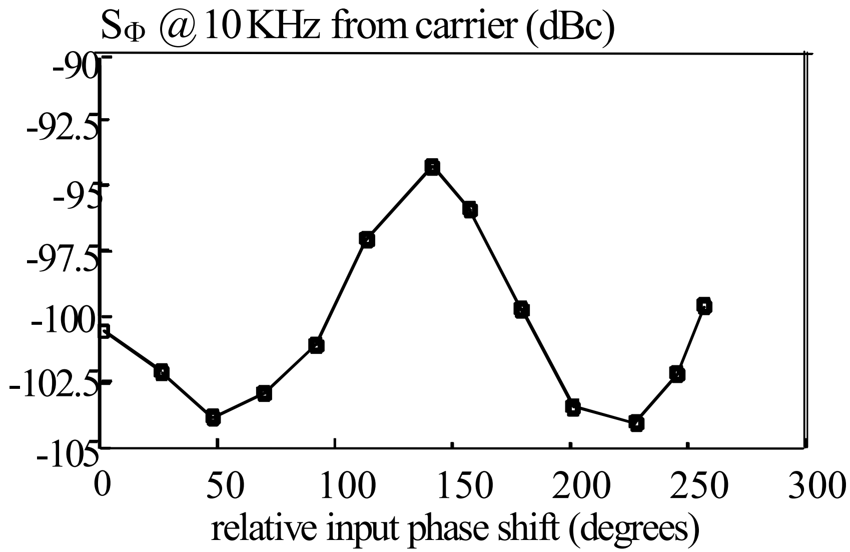


Linear representation

The analytical expression shows that $S_{\Delta\Phi out}$ varies periodically with θ_1.

Simulation of a complete circuit, including a nonlinear model of HEMT and accurate models of passive elements shows the same variation.

Finally, experimental results performed on a breadboard circuit, made with commercially available components give the following conclusion.



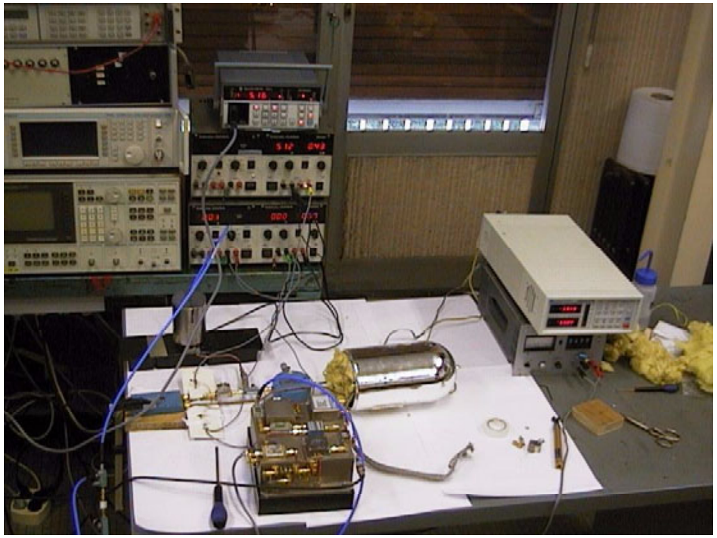
In this experiment :

- the loaded Q of the passive circuit is maintained constant
- the variable is θ_1, electrical length of the input resonator coupling line

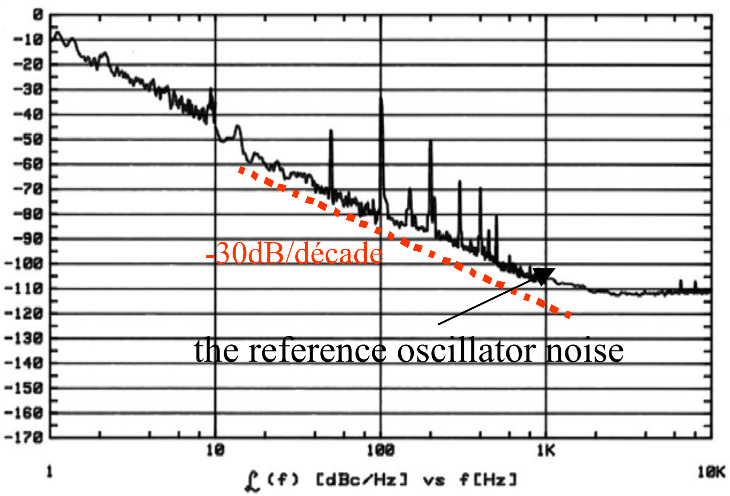
Following the modified Leeson formula, we have :

$$S_{\Delta\phi in} = K\left|1-\left[-\frac{1}{G_p}\right]e^{-2j\theta_1}\right|$$

where: - G_p is the transistor gain
- K is a coefficient independent of θ_1

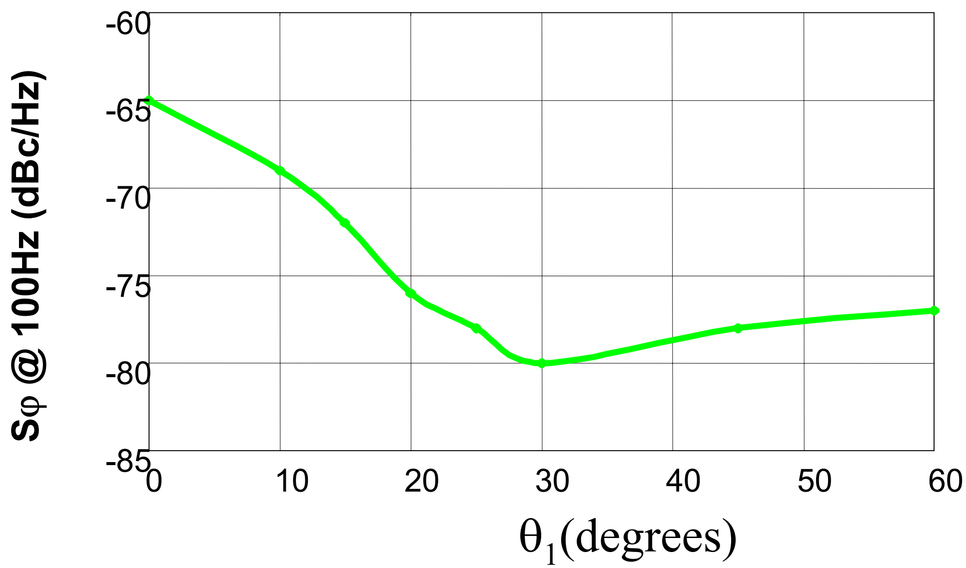


LPMO Phase noise measurement setup



↳S φ @ 100Hz=-80dBc

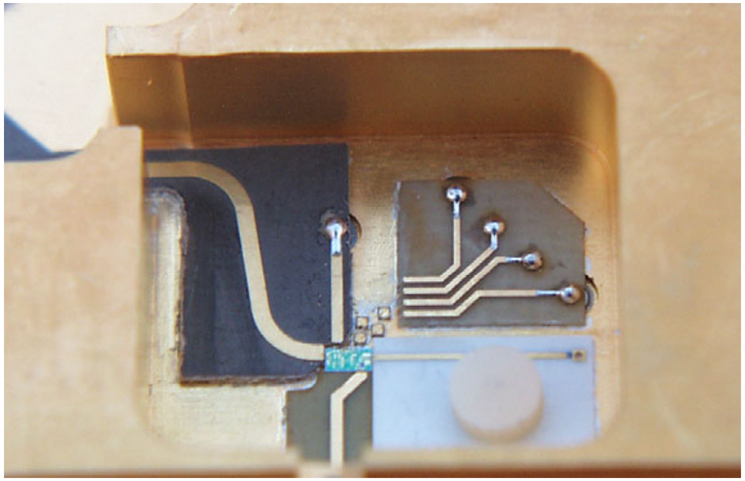
P_{oscil}=-6dBm et Fo=9.145GHz constants



Output phase noise spectral density versus the electrical length θ_1 of transmission line in the feedback path.

- MMIC-based oscillators for high volume production

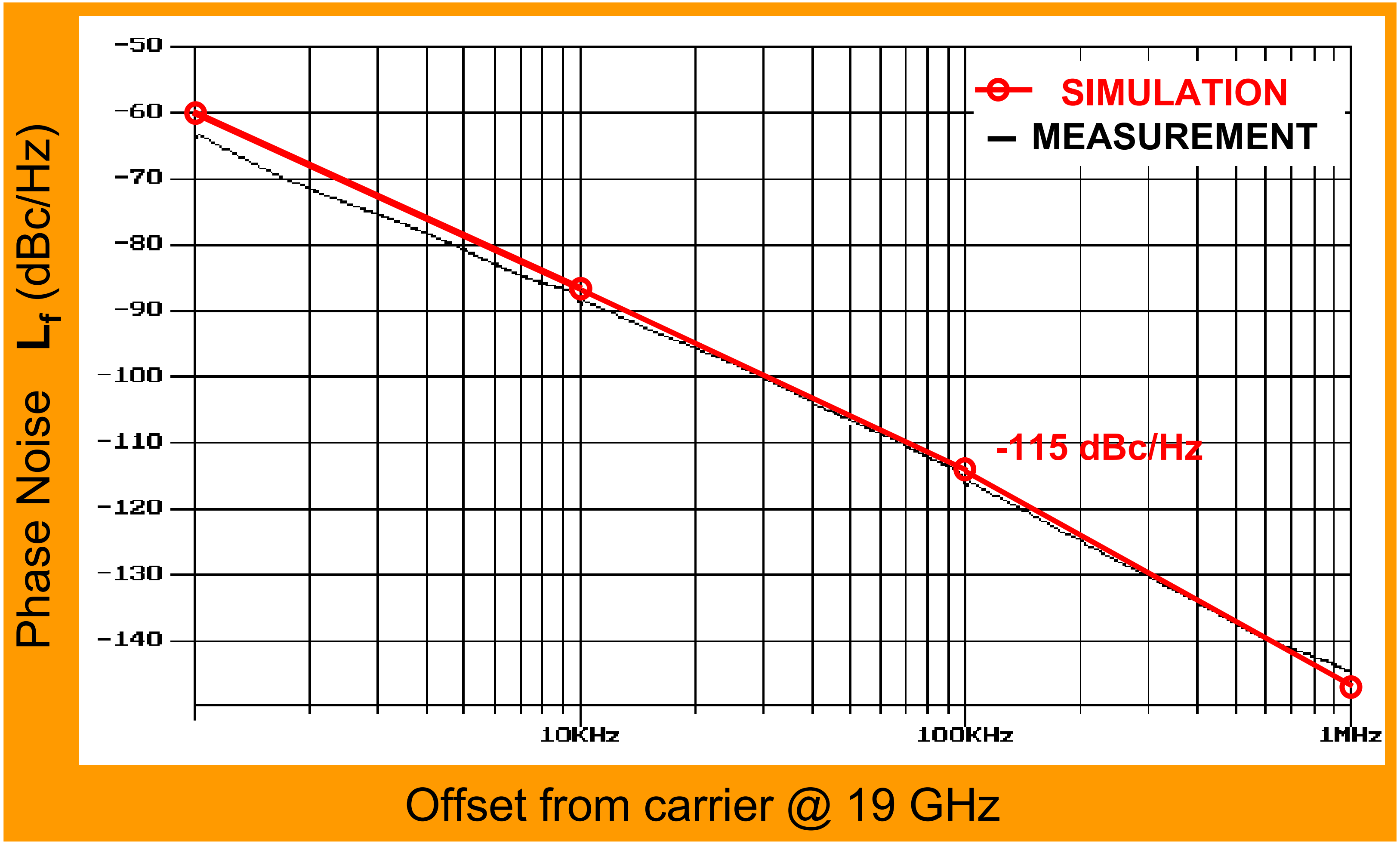
Circuits presented have been designed in an industrial environment using the tools presented in this talk.



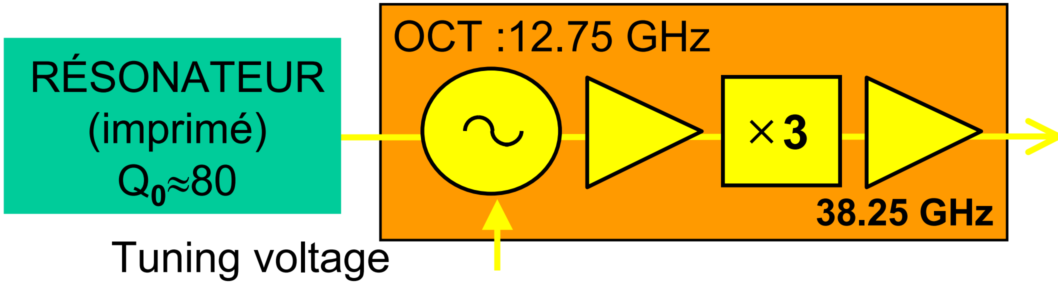
DRO with the external DR

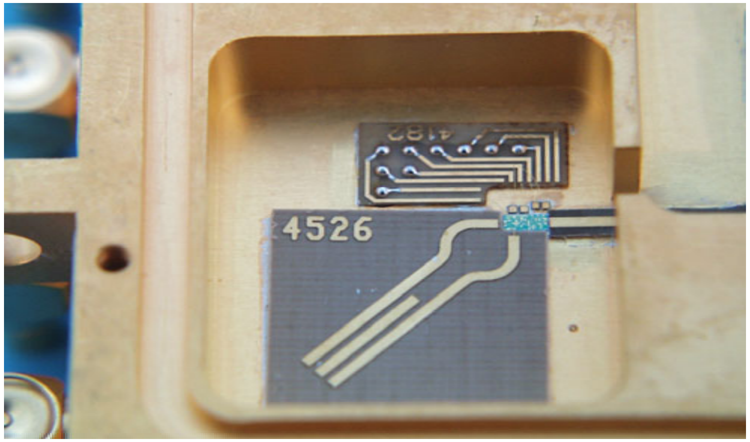
	measure	simulation
Auxiliary output power (dBm)	-12.4	-12.5
Oscillation frequency (GHz)	18.99	18.97

DRO PHASE NOISE SPECTRUM



VCO MMIC 38.25 GHZ (I)

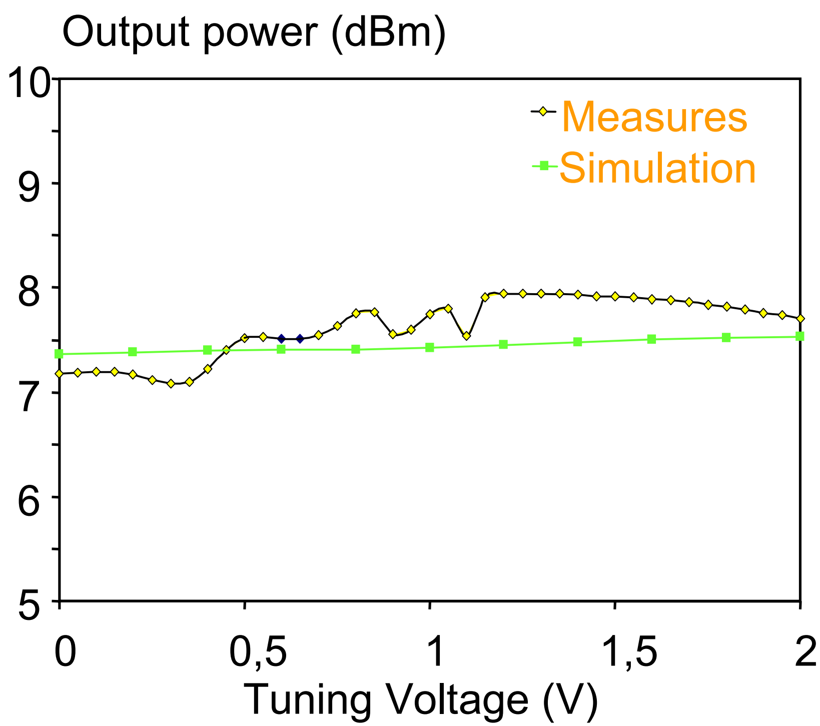


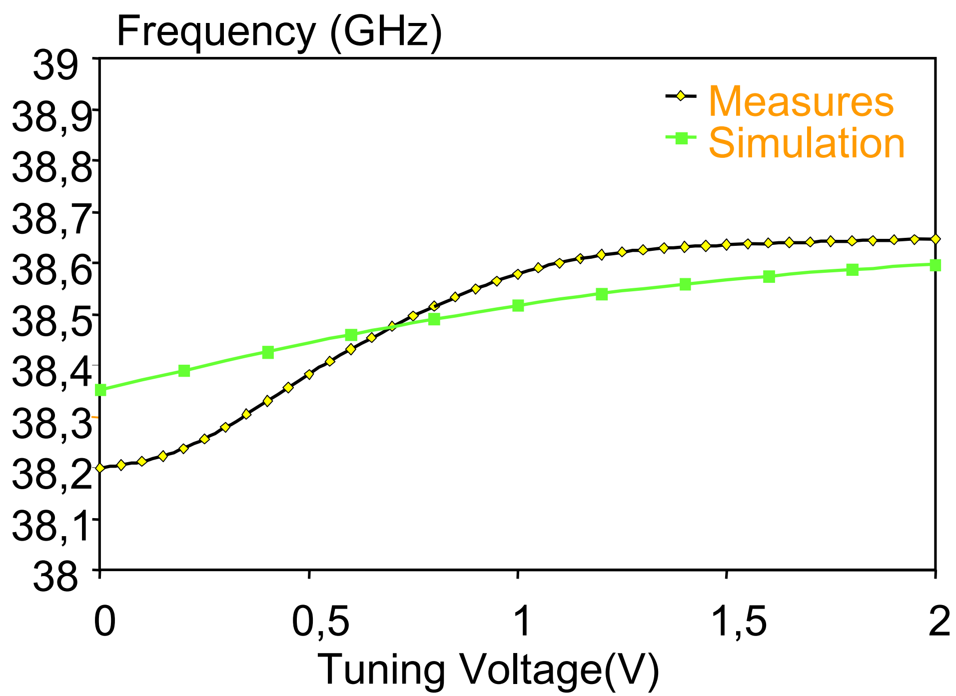


VCO with an external printed resonator

4 Transistors : 2×30 μm, 4×50 μm (×2), 4×75 μm

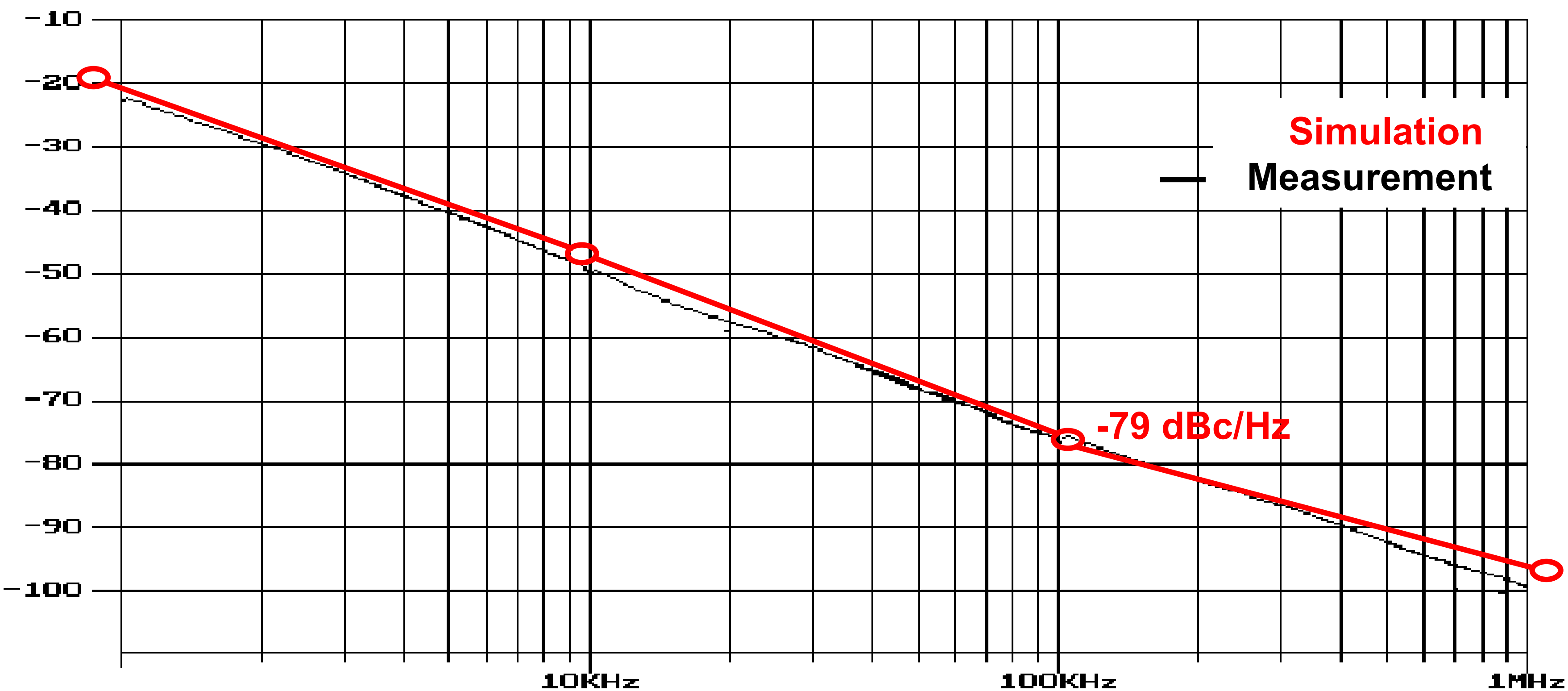
VCO MMIC 38.25 GHZ (II)





VCO MMIC 38.25 GHZ (III)

Phase noise $\mathcal{L}_f$ (dBc/Hz)



Offset from carrier @ 38.25 GHz

CONCLUSION

-The key-aspects of the design of low phase-noise RF, microwave, and millimetre-wave transistor-oscillators have been presented.

-The modern linear analysis of oscillator circuits has been detailed.

-Accurate non-linear models of transistors have been described.

-Modeling of low-frequency noise sources has been discussed .

-We have presented advanced design tools and methods, used in an industrial environment, making design reliable and leading to reproducible electrical characteristics.

-In conclusion, rapid developments of new methods, models, and tools, constrain the designer to systematically revisit his own design methodology.

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